Introduction to Complex Numbers Solutions Sheet

January 9, 2021

Exercise 1.

An example of a theoretically infinite process that can be counted by the natural numbers is counting the number of seconds starting from now, as in "1 second," "2 seconds," etc.

Exercise 2.

Typo, believe this question should have been written as $4 < x < 7$.

a) We can see that the natural number solutions to this equation are 5 and 6. Thus, using the set model, we can see that x is a solution if $x \in \{5, 6\}$.

b) With the number line, the solutions are the full tick marks at the natural numbers strictly to the right of 4 and to the left of 7, which include 5 and 6.

Exercise 3.

For a negative times a negative, we can think of speeds. For example, suppose your car c_0 is traveling along a road, and a car c_1 passes by that is traveling two times faster in the opposite direction. Then, suppose a car c_2 in an adjacent lane travels two times faster in again the opposite direction from c_1. The car c_2 travels four times faster than you, in the same direction.

For a positive times a positive, 2 baskets of 5 apples is 10 apples.

For a negative times a positive, we can think of loans. Suppose I owe you \$100. Then, suppose for some reason I now owe you two times more, so now I owe you \$200, which is money I have to take out of my paycheck in the future.

Exercise 4.

$$q1 = \frac{3}{8}$$
$$q2 = -\frac{6}{5}$$
$$q3 = \frac{5}{7}$$

Exercise 5.

Consider unit square $ABCD$. We have ABC is a right triangle, with AC a diagonal of the square. Let $l(AC)$ denote its length. Then, using the Pythagorean Theorem on ΔABC, we have $l(AC)^2 = l(AB)^2 + l(BC)^2 = 1^2 + 1^2 = 2$, from which we have $l(AC) = \sqrt{2}$.

Exercise 6.

We remember that $(xy)^2 = x^2y^2$. Using this, let $c = \sqrt{a} \cdot \sqrt{b}$ and $d = \sqrt{ab}$. We have $c^2 = (\sqrt{a} \cdot \sqrt{b})^2 = \sqrt{a}^2 \cdot \sqrt{b}^2 = ab = d^2$. As c and d are both nonnegative, as a and b are nonnegative here, we have $c = d$.

Exercise 7.

(1): $\sqrt{32} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$

(2): $\sqrt{200} = \sqrt{100}\sqrt{2} = 10\sqrt{2}$

(3): $\sqrt{324} = \sqrt{18^2} = 18$

(4): $\sqrt{68} = \sqrt{4}\sqrt{17} = 2\sqrt{17}$

(5): $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

Exercise 8.

(1): $\sqrt{-8} = \sqrt{4}\sqrt{-1}\sqrt{2} = 2i\sqrt{2}$

(2): $\sqrt{-36} = \sqrt{6}\sqrt{-1} = 6i$

(3): $\sqrt{-124} = \sqrt{4}\sqrt{-1}\sqrt{31} = 2i\sqrt{31}$

(4): $\sqrt{123464} = \sqrt{2^3 \cdot 11 \cdot 23 \cdot 61} = 2\sqrt{30866}$

Exercise 9.

(1): $i^3 = i^2 \cdot i = -i$

(2): $i^4 = (i^2)^2 = (-1)^2 = 1$

(3): $i^{73} = i^{72} \cdot i = (i^4)^{18} \cdot i = i$

Exercise 10.

$(3 + 4i) + (7 + 100i) = (3 + 7) + (4 + 100)i = 10 + 104i$

Exercise 11.

(1): $(104.4 + 1.9i) + (13 + 12 \cdot \frac{1}{2}i) = 117.4 + 7.9i$

(2): $(102.1 + 1.7i) + (1.6 + 3i) = 103.7 + 4.7i$

(3): $(x+4i)+(3x+yi)=24+11i$
$4x+i(4+y)=24+11i$
So, $4x=24$ and $(4+y)=11$
$\Rightarrow x=6$ and $y=7$

Exercise 12.

$$(3+4i)(7+100i)=3(7)+3(100i)+4i(7)+4i(100i)=-379+328i$$

Exercise 13.

Read

Exercise 14.

(1): $(3+2i)\cdot(4+3i)=6+17i$

(2): $(11+3i)\cdot(8+2i)=82+46i$

(3): $(14+2i)\cdot(3+2i)=38+34i$

(4): $4\cdot 13i=52i$