

COMBINATORICS III

MATH CIRCLE (INTERMEDIATE) 10/23/2011

1) How many ways are there to choose a vowel and a consonant from the word “RINGER”?

2) There are 10 students in a club. How many ways are there to elect a President, Vice-President, and a Secretary?

3) Suppose instead of three officers, the club from question 2 decided to have a committee of three people in charge. How many ways are there to pick such a committee?

Generally, suppose we have n objects and we want to choose k of the objects. There are $\frac{n!}{k!(n-k)!}$ ways to do so. This is called a combination and written $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. We often read $\binom{n}{k}$ as “ n choose k .”

4) There are 20 students in a gym class. How many ways are there to pick a basketball team (consisting of 5 people)?

5) How many ways are there to put 6 rooks on a chessboard so that they do not attack each other?

6) How many ways are there to split 14 people into 7 pairs?

7) We toss a dice 3 times. Among all the possible outcomes, how many have at least one occurrence of six?

8) How many 10-digit numbers have at least two equal digits?

9) Do 7-digit numbers having no digits 1 in their decimal representation constitute more than half of all 7-digit numbers?

Challenge 1) You have 6 identical balls and 4 boxes labelled A , B , C , and D . How many ways are there to place the balls into the boxes?

Challenge 2) You are making a bag consisting of 20 pieces of candy to give to your cousin for Halloween and can choose from 8 different types of candy. Assuming you give your cousin at least one of each piece of candy, how many different bags could you make?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes