

# Lesson 1: Weighings, Logic and Geometric Constructions

Konstantin Miagkov

## **Problem 1.**

You are given a balance scale and a weight of 1 gram. Can you use them to measure out 1 kilogram of sugar by using the scale at most 10 times?

## **Problem 2.**

There are 68 coins on the table, and any two coins weigh differently. Show how to determine the heaviest and the lightest coins using a balance scale at most 100 times.

We will explore the island of knights, knaves and spies, where there are three types of people: knights, who always tell the truth; knaves, who always lie; and spies, who can lie or tell the truth at will.

## **Problem 3.**

On the island of knights, knaves and spies, you come across three people. One wears blue, one wears red, and one wears green. You know that one is a knight, one is a knave, and one is a spy. “Who is the spy?” you ask.

- The man wearing blue says, “That man in red is the spy.”
- The man wearing red says, “No, the man in green is the spy.”
- The man wearing green says, “No, the man in red is in fact the spy.”

Who is the spy? Who is the knight and who is the knave?

## **Problem 4.**

There are 30 knights and knaves sitting at a round table. Each person has exactly one friend. Friend of a knight is always a knave, and friend of a knave is always a knight (friendship is mutual). You ask: “Is your friend sitting next to you?” and receive answers “Yes” from 15 people. How many other people could have possibly answered “Yes” as well?

**Problem 5.**

a) Given a segment  $AB$  on the plane, construct a point  $C$  such that  $ABC$  is an equilateral triangle using the ruler and compass.

b) Construct the midpoint of the segment  $AB$ .

**Problem 6.**

Given a point  $A$  and two rays out of it forming an angle  $\alpha$ , construct the angle bisector of  $\alpha$ .

**Problem 7.**

Let  $a_1, a_2, \dots$  be an infinite sequence of distinct positive integers, all of which are greater than 1. Show that there exist infinitely many  $i$  such that  $a_i > i$ .