Problem 1. (Textbook examples)
(a) We take a chessboard (8 \times 8) and remove two opposite corners. Show that one cannot tile
the remaining 62 squares with dominoes (\( \square \) or \( \mathcal{R} \)) completely and without overlaps.
(b) Consider a 2021 \times 2021 with a corner square removed. Can one tile the remaining squares
with linear trominos (\( \mathcal{T} \) or \( \mathcal{M} \)) completely and without overlaps?

Problem *2. Consider a \((4n + 2) \times (4n + 2)\) board.
(a) Show that the board cannot be tiled with T-tetrominoes (\( \mathcal{T} \) and its rotations) completely
and without overlaps.
(b) Show that the board cannot be tiled with L-tetrominoes (\( \mathcal{L} \) and its rotations) completely
and without overlaps.
(c) Show that the tilings in parts (a) and (b) are possible for a 4n \times 4n board.

Problem 3. The numbers from 1 to 25 are written in a 5 \times 5 board, one in each cell.
(a) Show that there exist two neighboring cells (i.e. sharing an edge) whose entries have distance
at least 3.
(b) Show that there exist two neighboring cells whose entries have distance at least 4.

Problem 4. A 2021 \times 2021 board is filled in with numbers from 1 to 2021, such that:
\begin{itemize}
  \item Every row and every column contains distinct numbers;
  \item The board is symmetric with respect to the main diagonal, i.e. the \((m, n)\)-th entry coincides
      with the \((n, m)\)-th entry.
\end{itemize}
Show that the numbers on the main diagonal are all distinct.

Problem 5. We are given a 2020 \times 2020 board, and we wish to paint half of its tiles in black, so
that there are no two adjacent black tiles (sharing an edge). In how many ways can this be done?
\textit{Hint: tiling can be useful!}

Problem *6. The tiles of a 4 \times 4 board are all colored in white initially. With every move, you
change the colors of all tiles in a row or in a column (from white to black and viceversa). Can you
ever reach a board with exactly 1 white tile? How about 2, 3, 4 and 5 white tiles?

Homework

Problem 1. Show that a 2021 \times 2022 board cannot be tiled with fish-shapes (\( \mathcal{A} \) and its rotations),
completely and without overlaps. \textit{Hint: the solution is very similar to that of 2 (a)}.

Problem 2. On an 8 \times 8 board, we wish to paint some of the tiles in red, such that no two red
tiles lie on the same diagonal. What is the greatest number of red tiles that we can paint? (Note:
in the language of chess, how many bishops can you put on the board such that no two attack
each other?)