Lesson 5: Quadratic Equations IV

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Problem 0.
Let $f(x) = ax^2 + bx + c$ be a quadratic equation with $a > 0$.
Show that f achieves its unique minimal value at $-b/(2a)$. In other words, show that for any $x \neq -b/(2a)$ we have

$$f(x) > f\left(\frac{-b}{2a}\right)$$

Show that if $a < 0$, then similarly f achieves its unique maximal value at $-b/(2a)$.

Problem 1.
Show that the equation $x^2 + px - 1$ has two distinct real roots for all values of p.

Problem 2.
a) Find a quadratic equation with roots $\sqrt{2}$ and $-\sqrt{7}$. Is it unique?

b) Find a quadratic equation with integer coefficients and a root $4 - \sqrt{7}$.

Problem 3.
a) Two real roots of the equation $ax^2 + bx + c = 0$ have difference 2020. What is the discriminant of this equation if $a = 1$?

b) Prove that the equation $ax^2 + 2bx + 4c$ has two roots.

c) What is the difference between them?

Problem 4.
Is it true that if $b > a + c > 0$, then the quadratic equation $ax^2 + bx + c = 0$ has two distinct real roots?

Hint: Look at $f(-1)$ and $f(1)$.

Problem 5.
All three coefficients of a quadratic equation are odd integers. Show that it cannot have a root of the form $1/n$, where n is an integer.