

COMBINATORICS II

MATH CIRCLE (INTERMEDIATE) 10/16/2011

If n is a natural number, then $n!$ (said n factorial) is the product $n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$. For example, $2! = 2$, $3! = 6$, and $4! = 24$. For convenience, we define $0! = 1$.

1) Simplify the expressions $10! \cdot 11$ and $n! \cdot (n+1)$. Calculate $\frac{100!}{98!}$ and $\frac{n!}{(n-1)!}$.

2) Prove that if p is a prime number, then $(p-1)!$ is not divisible by p .

3) How many ways are there to lay four balls, colored red, blue, black, and green, in a row? How many different “words” that can be obtained using rearranging the letters of “VECTOR”?

4) How many different words that can be obtained using rearranging the letters of “TRUST”? (Hint: Something different happens than in 3.) How about the word “CAR-AVAN”?

Generally, if we have n_1 of letter A_1 , n_2 of letter A_2 , . . . , and n_k of letter A_k , there are

$$\frac{(n_1 + \cdots + n_k)!}{n_1! \cdots n_k!}$$

different words formed using all of the letters.

5) How many different words that can be obtained using rearranging the letters of “CLOSENESS”? How about “MATHEMATICAL”?

6) There are 20 towns in a certain country, and every pair of them is connected by an air route. How many air routes are there?

7) How many diagonals are there in a convex n -gon?

8) How many ways are there to seat 13 people at a round table?

9) How many five-digit numbers have at least one even digit?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes