Functions Amendment

S. Hunter Gawboy

November 22th, 2020

Problem 4

Give a sequence of functions $\{f_n(x)\}$ for each case below:

- $f_n \geq f_{n+1}\ \forall n \in \mathbb{N}$
- $f_n \leq f_{n+1}\ \forall n \in \mathbb{N}$
- $f_n \leq f_{n+1}$ for n even and $f_n \geq f_{n+1}$ for n odd. $n \in \mathbb{N}$

Problem 5

Let $f_\delta(x) := f(x+\delta)$

Show $f_\delta \geq f\ \forall \delta > 0 \Rightarrow f$ is monotonic increasing and $f_\delta \leq f\ \forall \delta > 0 \Rightarrow f$ is monotonic decreasing.

Definition

We say $f(x)$ is bounded above(below) by M if $f(x) \leq M$ $(f(x) \geq M)$ $\forall x$.

Problem 6

Show $f(x) = \pm x^2$ is bounded

Problem 7

Show $f(x) = \frac{1}{x^3}$ over the open interval $(0, \infty)$ (positive reals) is bounded.
Can you find a domain for f such that f is not bounded?

This should tell you that a functions domain and codomain are important when defining that function; as, that function's properties can change depending on these sets.

Problem 8

Prove f is monotonic increasing or decreasing and bounded $\Rightarrow$ f has an asymptote.