

CONSTRUCTIONS IN PLANE GEOMETRY

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When performing constructions with a compass and straightedge, the basic allowable operations are:

- Drawing a line segment through two points with a straightedge.
- Extending a line segment to a line with a straightedge
- Drawing a circle with given center, and which passes through another given point.
- Finding the point(s) of intersection of two lines, two circles, or a line and a circle.

Warm-Up Problems.

- (1) Construct a line segment with length equal to the sum of the lengths of two given line segments.
- (2) Construct a triangle with given three sides.
- (3) Construct an angle congruent to a given one.
- (4) Construct the sum of two given angles.
- (5) Construct a triangle given:
 - (a) two sides and the angle between them.
 - (b) one side and both angles adjacent to it.
 - (c) two sides and the angle opposite to one of them.

Basic Constructions.

- (1) Bisect a given angle.
- (2) Construct two angles whose sum and difference are given.
- (3) Find the midpoint of a given line segment.
- (4) Erect a perpendicular to a given line from a given point on that line.
- (5) Drop a perpendicular from a given point to a given line (the point is not on the line).
- (6) Find the center of a given circle.
- (7) Construct a circle centered at a given point that touches a given line.
- (8) Construct a circle centered at a given point that touches a given circle.
- (9) Construct a line passing through a given point that touches a given circle.
- (10) Construct a line passing through a given point and parallel to a given line.

Harder Problems.

- (1) Construct a triangle if you know:
 - (a) its base, altitude, and one of the angles adjacent to the base.
 - (b) the three midpoints of its sides
 - (c) the lengths of two of its sides, and the median to the third side.
 - (d) two straight lines which contain angle bisectors, and the third vertex.
- (2) Construct the midpoint of a line segment:
 - (a) using only a compass.
 - (b) using a two-sided ruler whose width is smaller than the length of the line segment.
 - (c) using a two-sided ruler whose width is larger than the length of the line segment.
- (3) Construct a right triangle given:
 - (a) both of its legs.
 - (b) one of its legs and the adjacent acute angle.
 - (c) the hypotenuse and an acute angle.
 - (d) one of the legs and the hypotenuse.

- (4) Construct a line passing through a given point and cutting congruent segments on the sides of a given angle.
- (5) On a given line, find a point equidistant from two given points (outside the line).
- (6) Find a point equidistant from the three vertices of a given triangle.
- (7) On an given line intersecting the sides of a given angle, and a point equidistant from those sides.
- (8) Find a point equidistant from the three sides of a given triangle.

Challenge Problems.

- (1) Let A and B be two points on a line ℓ . Let C and D be two points on the same side of the line ℓ . Find point $N \in \ell$ so that the angles $\angle ANC$ and $\angle DNB$ are congruent.
- (2) Construct a triangle with a given base, an angle at that base, and the sum of the other two sides.
- (3) Given line segments with lengths a , b and c construct segments with length $\frac{ab}{c}$ and length \sqrt{ab} .
- (4) Construct a triangle given its perimeter and two of its angles.
- (5) Construct a line that touches two given circles. Note: consider various cases of mutual positions of the circles.
- (6) Construct a triangle given one of its sides, the opposite angle, and the altitude dropped to that side.
- (7) Given three points construct three circles that would touch each other at those points.

REGULAR POLYGONS

A regular n -gon is a polygon with n sides of equal length, and all angles between adjacent sides equal.

See how many of the regular polygons you can construct.

- (1) Equilateral triangle (easy)
- (2) Regular hexagon (almost as easy)
- (3) Square (medium)
- (4) Regular octagon (medium)
- (5) Regular 12-gon (medium)
- (6) Regular 15-gon (medium)
- (7) Regular pentagon (hard)
- (8) Regular 17-gon (very hard)