# Lesson 6: Functions and Quadratic Equations

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**Definition 1.**
A **function** from set $X$ to set $Y$ is a rule that for each element $x \in X$ determines an element from the set $Y$, usually denoted $f(x)$.

**Definition 2.**
Let $f, g : \mathbb{R} \to \mathbb{R}$ be functions. We write $f > g$ ($f \geq g$) if for every $x \in \mathbb{R}: \;\; f(x) > g(x)$ ($f(x) \geq g(x)$).

**Problem 1.**
Determine if one (or none) of $f \geq g$, $f \leq g$ holds:
**a)** $f(x) = 2x + 3$, $g(x) = 2x + 1$
**b)** $f(x) = 2x^2$, $g(x) = 3x^2$
**c)** $f(x) = x + 4$, $g(x) = x^2$
**d)** $f(x) = 2x + 1$, $g(x) = -x^2$

**Problem 2.**
A function $f : \mathbb{R} \to \mathbb{R}$ is called *even* if $f(x) = f(-x)$ for all $x \in \mathbb{R}$. Similarly, a function is called *odd* if $f(x) = -f(-x)$ for all $x$.
**a)** Find which of the following functions are odd, even or neither: $x \cdot |x|$; $|x+1| - |x-1|$; $|x + 1| + |x - 1|$; $3x - x^2$.

**b)** Show that any function from $\mathbb{R}$ to $\mathbb{R}$ can be uniquely written as a sum of an even and an odd function.

**Problem 3.**
Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(2x + 1) = 4x^2 + 14x + 7$.

**Problem 4.**
Five integers are written on the board – three coefficients of a quadratic equation and two roots in arbitrary order. After one of the numbers is erased, the numbers $2, 3, 4, -5$ are left. What number was erased?

**Problem 5.**
Let $ABCD$ be a quadrilateral such that there exists a circle tangent to all of its four sides. Such a quadrilateral is called *circumscribed.* Show that $AB + CD = AD + BC$.