HALLOWEEN MATH!

UCLA Olga Radko Math Circle 11/1/2020

REVIEW:

A) Roman Numerals

Recall that: I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1000

Recall Rule 1: if there are a few equal basic Roman numerals in a row, we add them. No more than three equal basic numerals in a row are allowed. For example, CC = 100 + 100 = 200, MMM = 1000 + 1000 + 1000 = 3000.

Recall Rule 2: if one or a few equal smaller basic Roman numerals are to the right of a greater one, we add them. No more than three smaller equal basic numerals are allowed. For example, LX = 50 + 10 = 60, CLX = 100 + 50 + 10 = 160,DXXI = 500 + 10 + 10 + 1 = 521.

Recall Rule 3: if a smaller basic Roman numeral is to the left of a greater one, we subtract them. It is not allowed to put a few smaller equal basic numerals to the left of a greater one. For example, XL = 50 - 10 = 40, CD = 500 - 100 = 400.

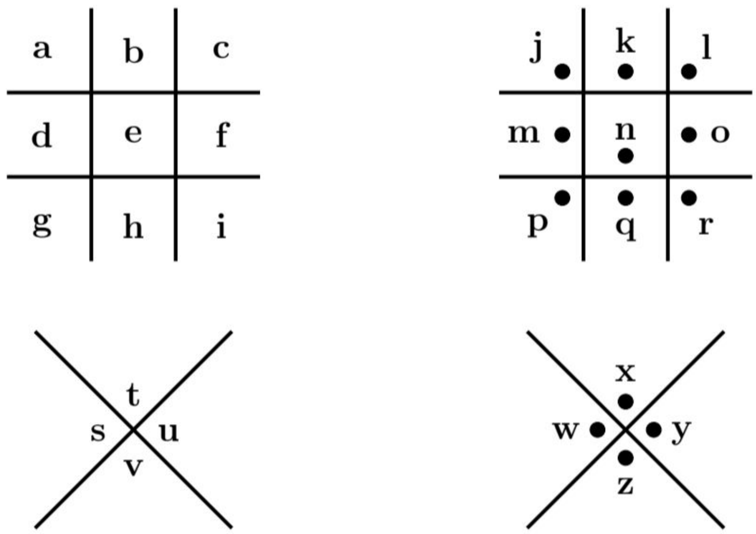
Recall Rule 4: subject to all other rules, one should write a Roman numeral in the simplest possible form. For example, the number DM is incorrect. It should be written as D.

Question 1: Correct the mistakes in the Roman numerals below. State which rule numbers are being violated for each.

• LXXX → XC; Rule 2	• MDCCCC → MCM; Rule 2
• DXXL → DXX; Rule 3	• CLL → CC; Rule 4
• DMM → MD; Rule 4	• XLCC → CLX; Rule 4
• DMDM → M; Rule 4	• XLCCXXIV → CLXXXIV; Rule 4

B) Pigpen Cipher

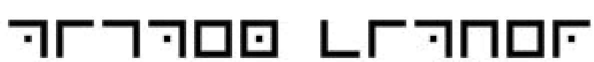
This cipher replaces letters with pigpen pictures from the table below.



Each letter is replaced by the frame surrounding it in the table. For example, *a* is replaced by  and *z* is replaced by  . The point is for the message to look like a child's drawing to an untrained eye.

Question 2: Use the pigpen cipher to encode the words

pigpen cipher

ANSWER: 

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C) Polybius Square

The encoding/decoding table on the left-hand side below is called the Polybius square. Invented in Ancient Greece about 2,200 years ago, the cipher was perfected by a Greek statesman, historian, and cryptographer, called Polybius.

	1	2	3	4	5
1	a	b	c	d	e
2	f	g	h	i/j	k
3	l	m	n	o	p
4	q	r	s	t	u
5	v	w	x	y	z

In the Polybius cipher, a letter is replaced by a two-digit number. For example, let us consider the letter r. Located at the intersection of the fourth row and the second column of the Polybius square, it is replaced by the number 42.

	1	2	3	4	5
1	a	b	c	d	e
2	f	g	h	i/j	k
3	l	m	n	o	p
4	q	r	s	t	u
5	v	w	x	y	z

To encode a letter: find it in the table. The first digit of the code for the letter is the number of the row in the table. The second digit is the number of the column.

Question 3: Use the Polybius square to encode the following message:

I love math!

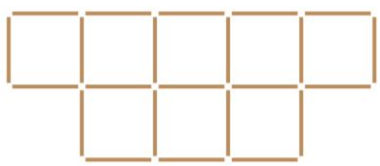
243134511532114423!

PROBLEMS:

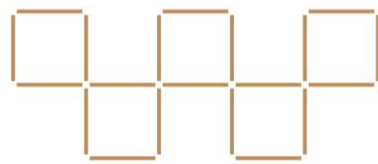
Question 1. Convert these Roman Numerals into numbers!

(a) MXXXI 1031 ________ **(b)** MMXX 2020 ________ **(c)** MMDCCCLXVII 2867 ________

Question 2. The picture below is made of craft sticks. Remove three of them so that the remaining sticks form five squares of equal size. Draw the five squares in the blank space to the right



Answer:



Question 3. Move two sticks to make the house look in the opposite direction. Draw the new house in the blank space to the right.

Answer:

Question 4. Your bag has 3 Tootsie Rolls, 8 candy corns, and 5 Jolly Ranchers.

(a) If you reach into the bag, what is the probability you will select a Tootsie Roll?

3/16

(b) How many candies must you pick to make sure that you have at least one piece of candy corn?

9

(c) How many candies must you pick to make sure that you have at least one piece of each kind of candy? (Hint: What is the worst case scenario?)

14

Question 5. Two groups of trick-or-treaters go into a neighborhood with a row of 51 houses. One group starts on the left and visits every 6th house while another group starts on the right and visits every 8th house.

(a) Will they ever meet?
Yes if you disregard the dimension of time.
No if you consider time.

(b) If so, at which house will they meet at?
Depending if you start at a "0th" house or the 1st house, answers may vary:
Starting at the first house:
Left: 1, 7, 13, **19**, 25, 31, 37, 43, 49
Right: 51, 43, 35, 27, **19**, 11, 3

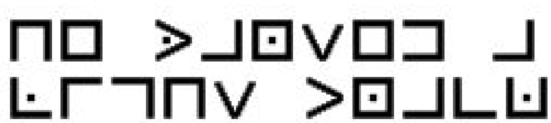
Starting from the "0th" house:
6, **12,** 18, 24, 30, **36**, 42, 48
44, **36**, 28, 20, **12**, 4

Question 6. Decipher this code using Polybius Square!

2311353554231131313452151533!

Happy Halloween!

Question 7. Decipher this Pigpen Riddle! Why did the vampire eat a light bulb?



HE WANTED A LIGHT SNACK

Answer: ______________________________

Question 8. Find all the solutions of the following cryptarithm. (This should be another challenge problem!

$$\begin{array}{rccccc} & G & H & O & S & T \\ + & G & H & O & S & T \\ \hline & H & O & U & S & E \end{array}$$

$G =$, $H =$, $O =$, $S =$, $T =$, $E =$, $U =$

Two solutions:

1) GHOST = 12403 and HOUSE = 24806
2) GHOST = 24903 and HOUSE = 49806

Thought process:
We have 10 digits (0,1,2,3,4,5,6,7,8,9) and seven letters. Each letter corresponds to an unique digit. Let's first try to think of any restrictions for each letter. So we can come up with:

Letter	Restriction	Reason
T	cannot be 0	or E would also be 0
E	has to be 2,4,6, or 8	T + T is even
S	has to be 0 if T <5 or 9 if T>=5	0+0=0, or if T >=5: 9+9+1 = 19
G	has to be 1,2,3, or 4	HOUSE is also a 5-digit number, so we cannot make it a 4 or 6-digit number

From our list of restrictions, we notice that S has only two possible values.
So let's split our work between when S =0 and when S=9. Then we will work from left to right and consider ALL possible values for the other letters.

If S=0:

Letter	Possible	Values	for	Letter
G	1	2	3	4
H	2	4	7	9
O	4	9	5	9--invalid!
U	8	8	0-invalid!	

S	0	0	0	0
T	3	3	--	--
E	6	6	--	--

Once we build the possible values of H,O, and U for each possible G value, we then try to find valid T,E pairings. Hence, we get our two solutions! But for completeness, please don't forget to do the same reasoning for when S = 9!

Challenge Problem

You came to a haunted house and saw 6 piles of coins on the table. The first one had only 1 coin, the next had 2, etc., so that the last had 6 coins. The witch in the haunted house tells you that you can take all of this gold if you can first make all of the piles equal. The only operation she allows you to perform is to take 2 coins from a huge bag and put each of them into a different pile on the table. Can you do this operation several times and make all the piles equal? If not, the witch is clearly tricking you!

(a) How many coins is there on the table when you first walk in the room?

21 coins

(b) As you continuously add two coins to the 6 piles, what do you notice consistently about the number of total coins?
We always have an odd number of total coins. So we notice that at least one pile is going to have a different amount of coins compared to the rest.

(c) Can you make all of the piles equal? Why or why not?

No, because $21 + 2n$ is not divisible by 6 as it is always odd.