

Geometry Homework Week 5

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1 Problems

1. If three non-intersecting circles have radical center O , prove that the points of contact of the six tangents from O to the circles all lie on one circle.
2. (Pitot's Theorem) Let $ABCD$ be a quadrilateral. Prove that a circle can be inscribed in the quadrilateral if and only if $AB + CD = BC + DA$.
3. Let ABC be a triangle with I_A, I_B, I_C its excenters. Prove that the triangle $I_A I_B I_C$ has orthocenter I and that triangle ABC is its orthic triangle.
4. (USAMO 2009/1) Given circles ω_1 and ω_2 intersecting at points X and Y , let ℓ_1 be a line through the center of ω_1 intersecting ω_2 at points P and Q and let ℓ_2 be a line through the center of ω_2 intersecting ω_1 at points R and S . Prove that if P, Q, R and S lie on a circle then the center of this circle lies on line XY .