SETS AND FUNCTIONS

JUNIOR CIRCLE 10/09/2011

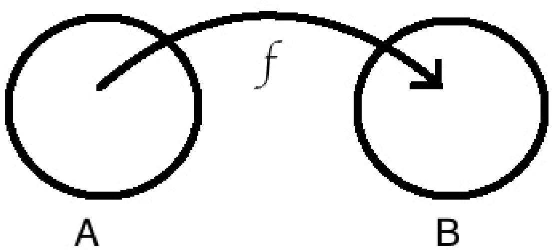
Let S be the set of all students in the Junior circle (who are in class today).

Let C be the set of all chairs in the Junior circle classroom.

Let I={Alyssa, Eric, Caitlin, Daniel, Rahlia, Kaley} be the set of instructors in Junior circle.

Let $\mathbb{N}=\{1, 2, 3, 4...\}$ be the set of natural numbers (these are the number we use when we count).

A *function* from a set A to a set B is a map or a rule that assigns to each element in A an element in B. The rule can be expressed in words, as a picture, by a table of values or as a formula.



Example 1. Define a function from the set of students S to the set of chairs C that assigns to each student the chair on which he or she is sitting.

Question: Do we know the value of this function for every student? If not, give an example when the function is not defined.

Example 2. Define a function from the set of students S to the set of instructors I that assigns to each student the instructor who sits at the same table.

Question: Do we know the value of this function for every student? If not, give an example when the function is not defined.

Example 3. Let $A = \{0, 1, 2, 3, 4, 5\}$ be a set. Define a function f from A to A by

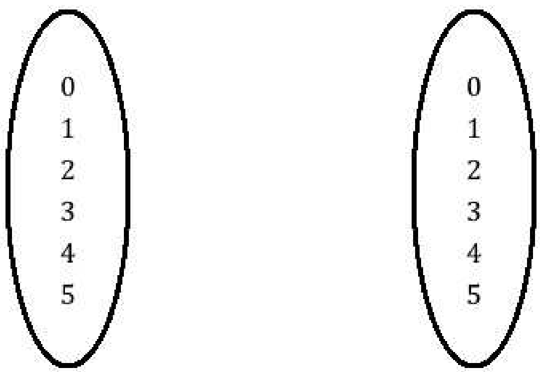
$$f(x) = 5 - x$$

That is, for every number x, the function converts it to $5 - x$.

Make a table of values for this function:

x	0	1	2	3	4	5
f(x)						

Draw an arrow from each number on the left to the corresponding number on the right:

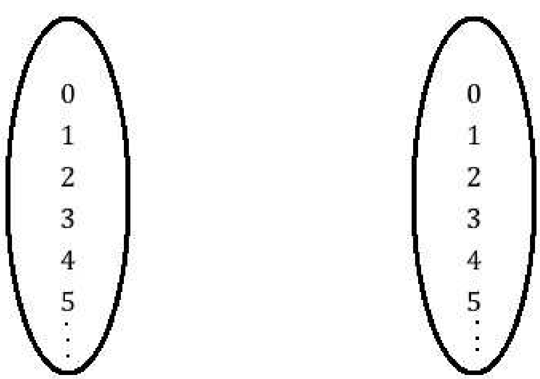


Example 4. Let $f : \mathbb{N} \to \mathbb{N}$ be a function that takes any natural number to 5.

Write down a formula describing this function:

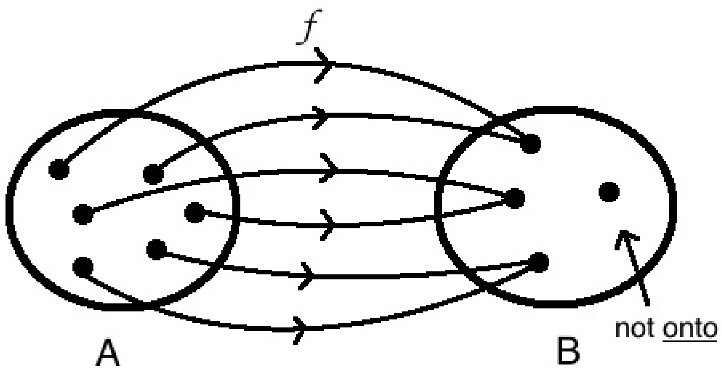
$$f(x) =$$

Make a picture of this using the map and arrow idea from Example 3.



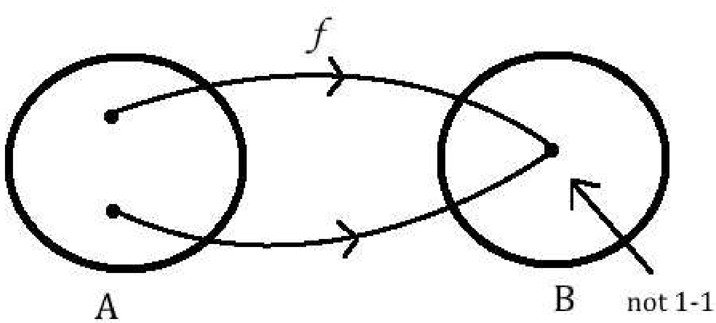
Definition 5. A function $f : A \to B$ from a set A to a set B is called *onto* if all the elements in B can be obtained as a result of the function.

Go back to Examples $1 - 4$ and label each function as "onto" or "not onto".



Definition 6. A function $f : A \to B$ from a set A to a set B is called $one - to - one$ if each of the values in B comes from at most 1 in A.

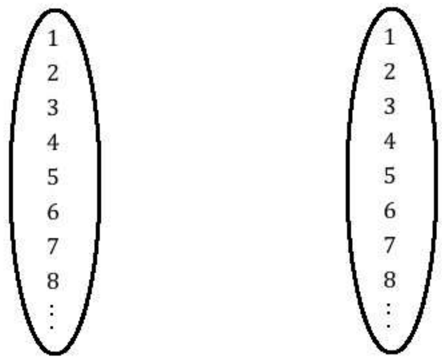
Go back to Examples $1 - 4$ and label each function as "$1 - 1$" or "not $1 - 1$".



(1) All functions $f : \mathbb{N} \to \mathbb{N}$

(a) $f(x) = x + 3$

(i) Make a picture of this function:



(ii) Can you get all numbers in $\mathbb{N}$ as a result of the function? If not, what are the numbers that you can not get?

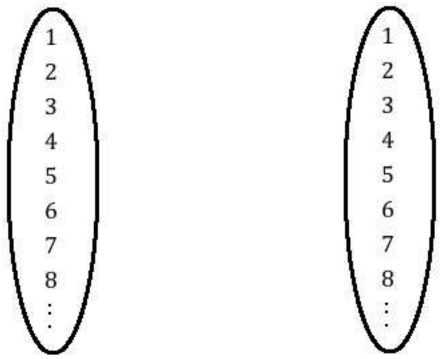
(iii) Is it true that any value can be obtained in *only* 1 way? If not, what values have the same output?

(iv) Is this function onto?

(v) Is this function $1 - 1$?

(b) $f(x) = \begin{cases} 1 & if\, x\, is\, even \\ 2 & if\, x\, is\, odd \end{cases}$

(i) Make a picture of this function:



(ii) Can you get all numbers in $\mathbb{N}$ as a result of the function? If not, what are the numbers that you can not get?

(iii) Is it true that any value can be obtained in *only* 1 way? If not, what values have the same output?

(iv) Is this function onto?

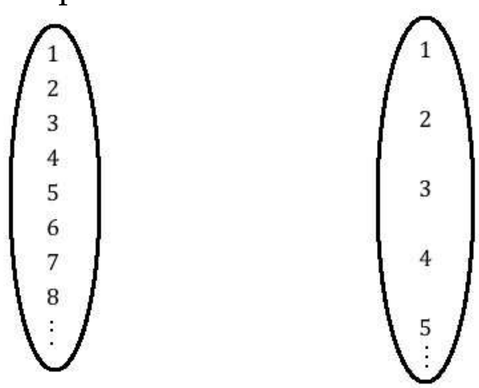
(v) Is this function $1-1$?

(c) $f(x)$ is such that:

x	1	2	3	4	5	6	7	8	$\cdots$
f(x)	1	1	2	2	3	3	4	4	$\cdots$

(i) Describe the function in words.

(ii) Make a picture of this function:



(iii) Can you get all numbers in $\mathbb{N}$ as a result of the function? If not, what are the numbers that you can not get?

(iv) Is it true that any value can be obtained in *only* 1 way? If now, what values have the same output?

(v) Is this function onto?

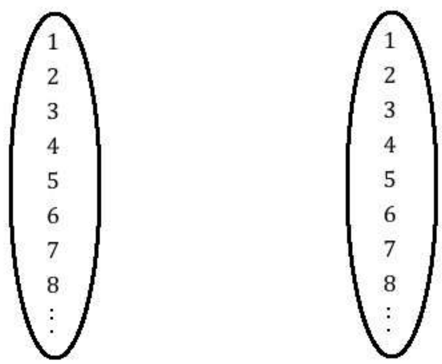
(vi) Is this function $1-1$?

(d) $f(x)$ is such that:

x	1	2	3	4	5	6	7	8	$\cdots$
f(x)	2	1	4	3	6	5	8	7	$\cdots$

(i) Describe the function in words.

(ii) Make a picture of this function:



(iii) Can you get all numbers in $\mathbb{N}$ as a result of the function? If not, what are the numbers that you can not get?

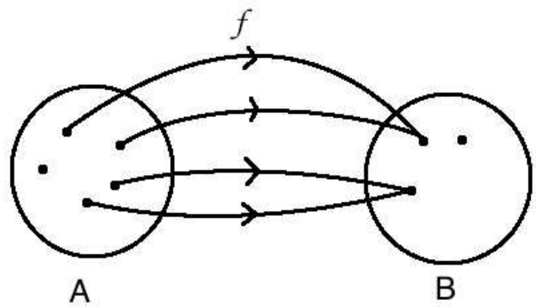
(iv) Is it true that any value can be obtained in *only* 1 way? If now, what values have the same output?

(v) Is this function onto?

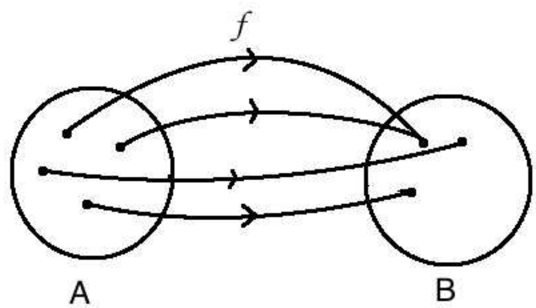
(vi) Is this function $1-1$?

(2) Use the pictures to answer the questions below.

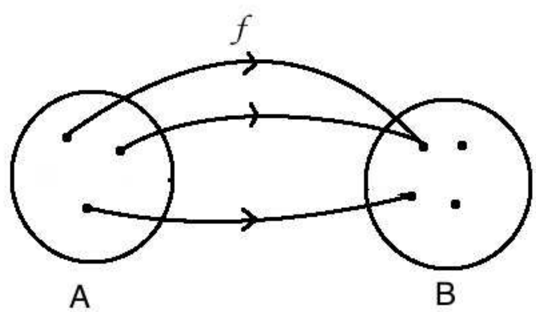
(a) Determine whether this function is defined. If yes, decide whether it is $1-1$ and whether it is onto.



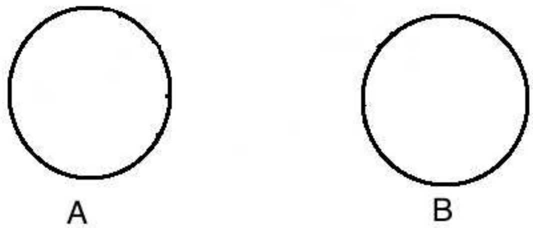
(b) Determine whether this function is defined. If yes, decide whether it is $1-1$ and whether it is onto.



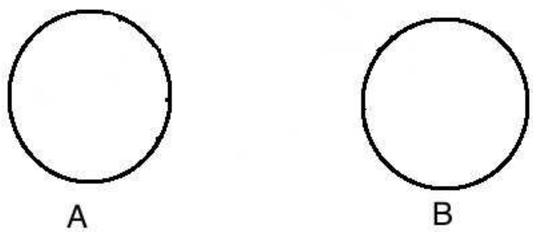
(c) Determine whether this function is defined. If yes, decide whether it is $1-1$ and whether it is onto.



(d) Make a picture showing a function that is onto, but not $1-1$.



(e) Make a picture showing a function that is $1-1$ and onto.



What can you say about the number of elements in both sets?

(3) Let A and B be two sets each consisting of n elements (where n is a finite number). Explain how we can construct a function $f : A \to B$ such that
- f is onto and
- f is $1-1$.

(*Hint*: number the elements of each set first. Make a picture)

(4) How do we compare infinite sets if we can not count the number of elements in them? What ideas can we use?

(5) Define a function from the set of odd numbers to the set of even numbers that is
- $1-1$
- onto

Make a conclusion.

(6) Define a function from the set of integers $\mathbb{Z} = \{... - 3, -2, -1, 0, 1, 2, 3...\}$ to the set of natural numbers $\mathbb{N} = \{0, 1, 2, 3, 4...\}$ that is
- $1-1$
- onto

Make a conclusion.