

Challenge Problem:

We have an infinite supply of yellow and green hats. The students sit in a line and each student can see all of the hats in front of them, but not their own or those behind them. The instructor starts at the back of the line and one by one asks each student which color hat they have. **The students know how the instructor is going to ask, and hence devise a plan to maximize the total number of correct guesses.**

For each number of students, please explain the students' plan and how many hats they are certain to guess correctly.

- 1) *There are 2 students and Student 1 is sitting behind Student 2. Since Student 1 is at the back of the line, the instructor asks Student 1, "What color hat do you have on?" Afterwards, proceeds to Student 2 and asks the same question.*

- a) *Is it possible for both Student 1 and Student 2 to correctly guess the color of their hats on the first try?*

They may guess correctly by sheer luck, but without a strategy it would be nearly impossible to guess correctly 100% of the time.

- b) *If it is not possible for both students to guess correctly for the first time, is there a strategy the two students can develop such that **at least one student** guesses their hat correctly?*

Have the first person give a "hint" for the people in front of them as to what color hats they have on. The students can devise a plan such focuses on the number of green hats, for example. If the number of green hats is 0 or an even number, the first person will say "Green." If the number of green hats is an odd number, the first person will say "Yellow."

- c) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

1

- 2) *There are 3 students. Student 3 sits in front of Student 2 and Student 1 is at the back of the line.*

- a) *Is it possible to expand the strategy developed in the 2-student scenario for the three students? If not, try to revise the strategy.*

Same strategy as above. For example, suppose the order of hats for the 1st, 2nd, and 3rd students was Yellow, Yellow, Green, respectively. 1st person sees a yellow and green hat and says "Yellow." This tells the 2nd and 3rd persons that the number of green hats is an odd number. In this case it must be 1. The 2nd person sees a green hat and thus says "Yellow." Indicating to the 3rd person that they must have the Green Hat.

- b) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

2

3) *There are 4 students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

3

4) *There are 5 students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

4

5) *There are 6 students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

5

6) *There are 7 students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

6

Are you starting to see a pattern?

7) *There are n students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

$n-1$
