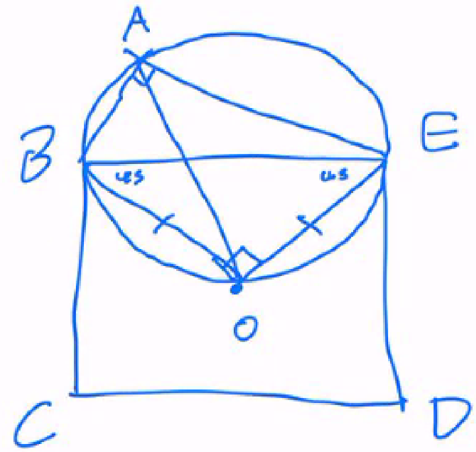
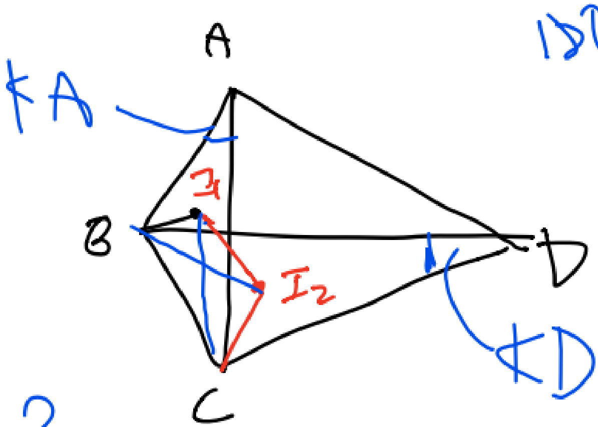
Problem 1.36. Let $ABCDE$ be a convex pentagon such that $BCDE$ is a square with center O and $\angle A = 90^\circ$. Prove that $\overline{AO}$ bisects $\angle BAE$. **Hints**: 18 115 **Sol:** p.241



ABOE cyclic

Problem 1.38. In cyclic quadrilateral $ABCD$, let I_1 and I_2 denote the incenters of $\triangle ABC$ and $\triangle DBC$, respectively. Prove that I_1I_2BC is cyclic. **Hints**: 684 569

$180 - \frac{B}{2} - \frac{C}{2}$,,



$\angle BI_1C \stackrel{?}{=} \angle BI_2C$

$BI_1C \to \angle BAC = \angle BDC \leftarrow \angle BI_2C$

$\angle BI_1C = 90 + \frac{\angle A}{2} = 90 + \frac{\angle D}{2} = \angle BI_2C$

Example 1.35 (Shortlist 2010/G1). Let ABC be an acute triangle with D, E, F the feet of the altitudes lying on $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ respectively. One of the intersection points of the line EF and the circumcircle is P. The lines BP and DF meet at point Q. Prove that $AP = AQ$.

WTS $\angle AP_2Q_2 = \angle C = \angle AQ_2P_2$

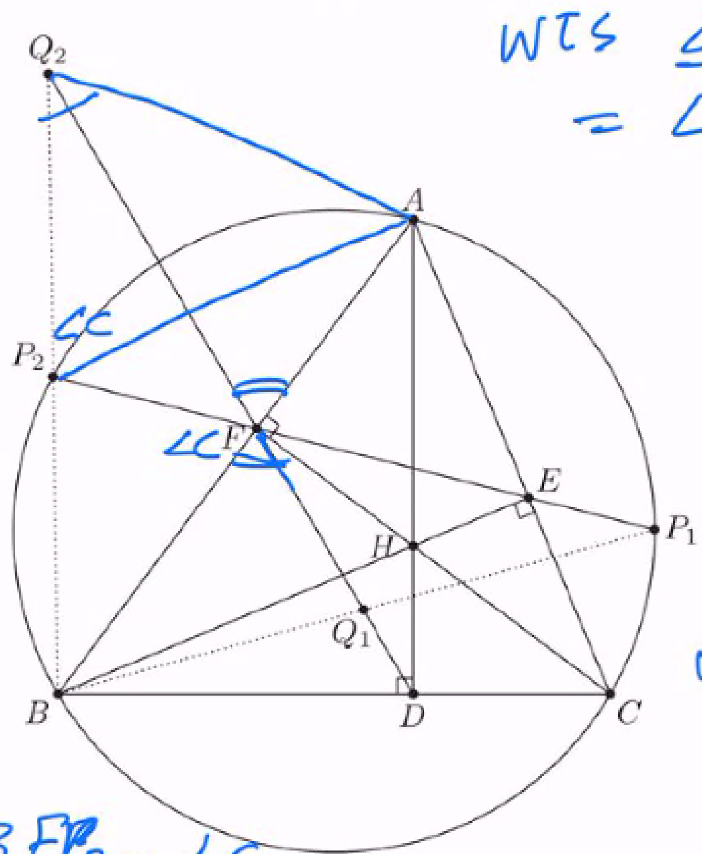
$\angle AFQ_2 = \angle C$

Using AFDC cyclic

$\angle QFD = \angle C = \angle AFQ_2$

$\therefore AQ_2P_2F$ cyclic

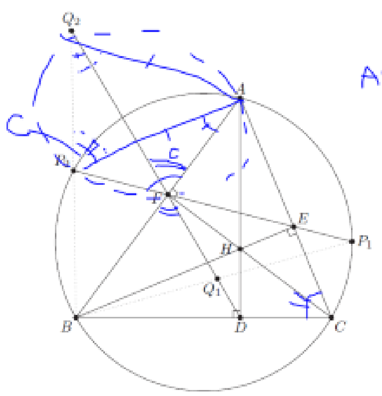
$\to \angle AQ_2P_2 = \angle BFP_2 = \angle C$



BFEC cyclic

$\to \angle AFE = \angle BFP_2 = \angle C$

WTS AQ_2P_2F cyclic.



AFP = 180 − C

180 − C

$\measuredangle QPA = \measuredangle C \overset{?}{=} \measuredangle PQA$

- $\measuredangle BFA = 180 - C$
- Cyclicity