Problem 1.36. Let $ABCDE$ be a convex pentagon such that $BCDE$ is a square with center $O$ and $\angle A = 90^\circ$. Prove that $AO$ bisects $\angle BAE$. **Hints:** 18 115 **Sol:** p.241

Problem 1.38. In cyclic quadrilateral $ABCD$, let $I_1$ and $I_2$ denote the incenters of $\triangle ABC$ and $\triangle DBC$, respectively. Prove that $I_1I_2BC$ is cyclic. **Hints:** 684 569

$\angle B_{I_1}C = \angle B_{I_2}C$

$\angle B_{I_1}C = \angle BAC = \angle BDC = \angle B_{I_2}C$

$\angle B_{I_1}C = 90 + \frac{\angle A}{2} = 90 + \frac{\angle D}{2} = \angle B_{I_2}C$
**Example 1.35 (Shortlist 2010/G1).** Let \( \triangle ABC \) be an acute triangle with \( D, E, F \) the feet of the altitudes lying on \( BC, CA, AB \) respectively. One of the intersection points of the line \( EF \) and the circumcircle is \( P \). The lines \( BP \) and \( DF \) meet at point \( Q \). Prove that \( AP = AQ \).

\[ \measuredangle AP_2Q_2 = \measuredangle C = \measuredangle AQ_2P_2 \]

\[ \triangle APD \text{ and } \triangle CPD \text{ are cyclic.} \]

\[ \measuredangle APD = \measuredangle C = \measuredangle CPD \]

\[ \therefore \triangle DPQ \text{ is isosceles.} \]

\[ \measuredangle DPQ = \measuredangle DQP \]

\[ \therefore \measuredangle DPQ = \frac{180 - \measuredangle C}{2} \]

\[ \measuredangle BFA = 180 - \measuredangle C \]

\[ \measuredangle BFA = \measuredangle APQ \]

\[ \text{Cyclicity} \]