

PARITY II

MATH CIRCLE (INTERMEDIATE) 10/02/2011

1) Seven gears are placed on a plane, arranged in a circular chain. Can all the gears rotate simultaneously?

2) The product of 22 integers (i.e. whole numbers) is equal to 1. Show that their sum cannot be 0.

3) The numbers 1 through 10 are written in a row. Can the signs “+” or “-” be placed between them, so that the value of the resulting expression is 0?

4) Can a 5×5 chessboard be covered by 1×2 dominoes?

5) Can an ordinary 8×8 chessboard be covered with 1×2 dominoes so that only squares $a1$ and $h8$ remain uncovered? (Note that $a1$ is the bottom left square of the board while $h8$ is the upper right square.)

6) Can a knight start at square $a1$ of a chessboard, and go to square $h8$, visiting each of the squares exactly once on the way?

7) There are 100 soldiers in a detachment, and every evening three of them are on duty. Can it happen that after a certain period of time each soldier has shared duty with every other soldier exactly once?

8) A grasshopper jumps along a line. His first jump takes him 1 cm, his second 2 cm, and so on. Each jump can take him to the right or to the left. Show that after 2010 jumps the grasshopper cannot return to the point at which he started.

9) Three hockey pucks, A , B , and C , lie on a playing field. A hockey player hits one of them in such a way that it passes between the other two. He does this 25 times. Can he return the three pucks to their starting points?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes