A function $f(x)$ is called \textit{convex} if for any $x_1$ and $x_2$ in its domain and for any $0 < \alpha < 1$,

$$f(\alpha x_1 + (1-\alpha)x_2) < \alpha f(x_1) + (1-\alpha)f(x_2).$$  \hfill (1)

\textbf{Problem 1} \textit{Give a geometric interpretation to formula (1).}

\textbf{Problem 2} \textit{Prove that for a linear function $f(x) = bx + c$, $f(\alpha x_1 + (1-\alpha)x_2) = \alpha f(x_1) + (1-\alpha)f(x_2)$ for any value of the parameter $\alpha$.}
Problem 3 Prove that $f(x) = ax^2 + bx + c$ is convex for $a > 0$.

The value $\hat{x}$ is called a minimum of a function $f(x)$ if $f(\hat{x}) \leq f(x)$ for every $x$ in the function’s domain.

Problem 4 Sketch the graph of a function having two minima.
**Problem 5** *The function* $f(x)$ *is convex. Prove that it can have at most one minimum.*

**Problem 6** *Find the minimum of the function* $f(x) = ax^2 + bx + c$, $a > 0$. *Prove that it is indeed a minimum. What is the value of the function at the point?*

**Problem 7** *Find the minimum of the function* $f(x) = (x - a_1)^2 + (x - a_2)^2 + \ldots + (x - a_n)^2$. 