LAMC Advanced Circle **February 12, 2017**

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A function $f(x)$ is called *convex* if for any x_1 and x_2 in its domain and for any $0 < \alpha < 1$,

$$f(\alpha x_1 + (1-\alpha)x_2) < \alpha f(x_1) + (1-\alpha)f(x_2). \tag{1}$$

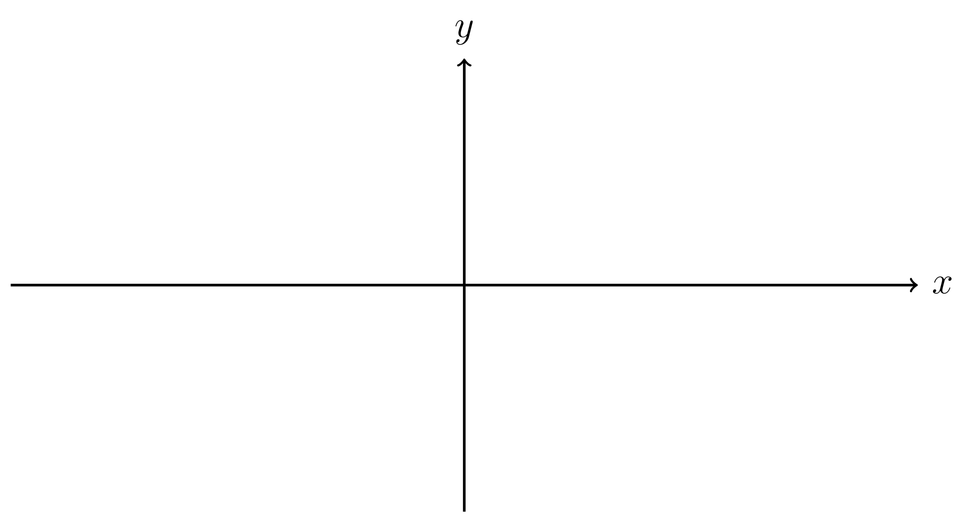
Problem 1 *Give a geometric interpretation to formula (1).*

Problem 2 *Prove that for a linear function* $f(x) = bx + c$, $f(\alpha x_1 + (1-\alpha)x_2) = \alpha f(x_1) + (1-\alpha)f(x_2)$ *for any value of the parameter* α.

Problem 3 *Prove that* $f(x) = ax^2 + bx + c$ *is convex for* $a > 0$*.*

The value $\hat{x}$ is called a *minimum* of a function $f(x)$ if $f(\hat{x}) \leq f(x)$ for every x in the function's domain.

Problem 4 *Sketch the graph of a function having two minima.*



Problem 5 *The function $f(x)$ is convex. Prove that it can have at most one minimum.*

Problem 6 *Find the minimum of the function $f(x) = ax^2 + bx + c$, $a > 0$. Prove that it is indeed a minimum. What is the value of the function at the point?*

Problem 7 *Find the minimum of the function $f(x) = (x - a_1)^2 + (x - a_2)^2 + \ldots + (x - a_n)^2$.*