Problem 1. How many positive integers \( n \leq 30,000 \) satisfy \( 2 \mid n \), \( 3 \mid n - 1 \) and \( 5 \mid n - 2 \)?

Problem 2.
(a) Find all pairs of integers \((a, b)\) such that \( 3a + 5b = 1 \).
(b) For what integers \( c \) does the equation \( 3a + 5b = c \) have a solution in integers \( a, b \)?
(c) Find all pairs of integers \((a, b)\) such that \( 3a + 5b = 4 \).

Problem 3.
(a) Find all pairs of integers \((a, b)\) such that \( a^2 + 5b^2 = 2^{2020} \).
(b) Find all pairs of integers \((a, b)\) such that \( a^3 + 5b^3 = 7^{2020} \).

Problem 4. Find all solutions in positive integers to the equation \( a^2 + b^4 + c^{10} = 3 \cdot 2^{2020} \).

Problem 5. Find all integer solutions to the equation \( a^3 + 3b^3 + 9c^3 = 0 \).

Problem 6. Find all pairs of integers \((a, b)\) such that \( 4a^2 - b^3 = 1 \).

Problem 7. Below, \( n \) denotes a positive integer.
(a) Find all perfect cubes of the form \( 2^n - 1 \).
(b) Find all perfect cubes of the form \( 3^n - 1 \).

Problem 8.
(a) Find all pairs of positive integers \((a, b)\) such that \( 2^a + 1 = 3^b \).
(b) Find all pairs of positive integers \((a, b)\) such that \( 2^a - 1 = 3^b \).
(c) Find all solutions in positive integers to the equation \( 2^a + 3^b = 5^c \) with \( a \geq 2 \).

Problem 9. Find all solutions in nonnegative integers to the equation \( 2^{2^a} + 2^{2^b} = 2^{2^c} \).

Problem *10. Show that the equation \( 2^a - a^3 = b! \) has only finitely many solutions in positive integers \( a, b \).

Problem *11. Show that \( 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \) is never an integer for \( n \geq 2 \).

Problem *12. Find all prime numbers of the form \( n^8 + 2^{2^n+2} \), where \( n \) is a positive integer.
Problem 1. Show that there are infinitely many integer values \( n \), such that \( 7 \mid 2^n - 1 \) and \( 11 \mid 3^n - 4 \).

Problem 2. Solve the equation \( a^2 + 2b^2 = 11 \cdot 2^n \), where \( a, b, n \) are positive integers.

Problem 3. Show that there are infinitely many positive integers \( n \) such that the expression (i.e. polynomial) \( x^n + n \) can be factored (i.e. written as a product of other non-constant polynomials with integer coefficients; for instance, \( x^{27} + 27 = (x^9 + 3)(x^{18} - 3x^9 + 9) \)).

Homework 2

Problem 4. Find all perfect squares of the form \( 5^n + 4 \).

Problem 5. Solve the equation \( 7^a + 5 = 6^b \), where \( a \) and \( b \) are non-negative integers.

Problem 6. Show that the equation \( 2^a + 1 = 11^b \) does not have any solutions in positive integers \( a \) and \( b \).