From Fraction to Decimal

1. Find a common multiple for each of the following sets of numbers. Then find the **least** common multiple (LCM), if different:

   (a) 10 and 14
   - 70

   (b) 8 and 50
   - 200

   (c) 37 and 111
   - 111

   (d) any two consecutive numbers n and $n + 1$
   - $n \times (n + 1)$

   (e) 2, 3, and 5
   - 30

   (f) $2 \times 2 \times 3$, 3×5, and $2 \times 2 \times 5$
   - $2 \times 2 \times 3 \times 5 = 60$

2. For each of the following pairs of fractions, compare them using $>$, $<$, or $=$. You may need to rewrite them using a common denominator, but however you do it, be ready to explain your reasoning. **(NO CALCULATORS!)**

   (a) $\frac{3}{10}, \frac{5}{14}$
   - $3/10 = 21/70 < 25/70 = 5/14$

   (b) $\frac{43}{44}, \frac{43}{45}$
   - $43/44 > 43/45$ (same numerator, smaller denominator => bigger)

3. Put the fractions in increasing order by finding a common denominator. (If two fractions are equal, they can go in either order.) **(NO CALCULATORS!)**

   (a) $\frac{9}{12}, \frac{11}{15}, \frac{15}{20}$
   - $9/12 = 45/60$, $11/15 = 44/60$, $15/20 = 45/60 => 11/15 < 9/12 = 15/20$

4. Mr. Ganzfeld owns a number of chickens and cows. His collection of animals has 22 heads and 62 legs. How many chickens does he have?

   - 13 chickens (and 9 cows)

5. If you have two fractions (which might be any fractions at all), can you always find a common denominator for them that's...

   (a) less than 1,000,000?
   - no; for instance 1/1,000,001 and any other fraction won't work

(b) greater than 1,000,000?

- yes–given any common denominator, multiply it by 1,000,000 and it will still be one, and now $> 1{,}000{,}000$

(c) even?

- yes–given any common denominator, multiply it by 2 and it will still be one and now even

(d) odd?

- no; for instance 1/2 and any other fraction won't work

(e) a power of two?

- no; for instance 1/3 and any other fraction won't work

(f) a square number?

- yes–given any common denominator, multiply it by itself and it will still be one and now square

For each property, if you said "yes, you can always find one," explain why. If you said, "no, you can't always find one," give an example of two fractions whose common denominator doesn't have the property.

6. Suppose your friend has two secret fractions $\frac{a}{b}$ and $\frac{c}{d}$, where you don't know what a, b, c, d are (they can be any whole numbers). If you know that

$$\frac{a}{b} > \frac{c}{d},$$

does this tell you anything about the relationship ($>$, $<$, or $=$) between the products...

(a) ab and cd?

- $ab \neq cd$, but it could be larger or smaller. For instance $2/3 > 1/3$ and $2*3 > 1*3$, but $1/2 > 2/5$ and $1*2 < 2*5$.

(b) ac and bd?

- same as before: they can't be equal but either of $>$, $<$ is possible. For instance, $1/2 > 1/3$ and $1*1 < 2*3$, but $5/2 > 1/2$ and $5*1 > 2*2$.

(c) ad and bc?

- It's guaranteed that $ad > bc$ (you can see this by rewriting a/b $>$ c/d using the common denominator bd.)

(If you're not sure, try some examples for the two fractions! Previous problems can help with this.)

7. It's easy to convert a fraction to a decimal when the denominator is already a power of 10. For instance,

$$39/10 = 3.9, \text{ and } 12345/1000 = 12.345.$$

We can convert other fractions to decimals by long division. For each of the following sets of numbers, put them in increasing order by writing them as decimals using long division:

(a) $\frac{58}{10}, \frac{572}{100}, 6$

- $572/100 = 5.72 < 58/10 = 5.8 < 6$

(b) $\frac{2}{10}, \frac{1}{5}$

- $.2 = .2$

(c) $\frac{667}{100}, \frac{3333}{500}$

- $3333/500 = 6.666 < 6.67 = 667/100$

(d) $\frac{7}{4}, \frac{42}{25}$

- $7/4 = 1.75 > 1.68 = 42/25$

(e) $\frac{36}{5}, 7, \frac{27}{4}, \frac{135}{20}$

- $36/5 = 7.2 > 7 > 27/4 = 6.75$ =135/20

8. Here are some more fractions. What happens when you use long division to write them as decimals?

   (a) $\frac{1}{3}$

   - .(3)

   (b) $\frac{5}{9}$

   - .(5)

   (c) $\frac{2}{3}$

   - .(6)

   (d) $\frac{1}{7}$

   - .(142857)

   (e) $\frac{16}{99}$

   - .(16)

9. The denominators of fractions in exercise 7 all had a certain property in common, let's call it Property X, that the denominators in problem 8 did not have. Here are some more numbers with Property X:

   - 20, 5, 4, 1000, 16, 50, 2, 800, 125, 10

   The numbers in the next set do **NOT** have Property X:

   - 30, 3, 9, 1001, 36, 55, 7, 700, 130, 15

   Can you figure out what Property X is? (Note: Try to express Property X as something to do with the number itself, not something to do with fractions that have that number as a denominator.)

   - Numbers with property X have no prime factors other than 2 and 5.

10. What is the difference between decimal representations of fractions whose denominator has Property X, and fractions whose denominator doesn't have Property X? (You can assume for the sake of this question that the fractions are reduced. How does it change things if they might not be reduced?)

    - Fractions whose denominators have Property X have terminating decimals; if the denominator does not have Property X, the decimal goes on forever (repeats itself).

11. What do you get when you multiply...

    (a) $\frac{1}{2} \times \frac{2}{3}$

    - 1/3

    (b) $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

    - 1/4

    (c) $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$

    - 1/5

    (d) $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \cdots \times \frac{n}{n+1}$

    - 1/(n+1)

12. Ms. Smith didn't have any cash, so she went to the bank with a check she had. A bank teller who wasn't very good at his job switched the dollars and cents when he cashed the check for her. (For example, if the amount on the check was \$27.50, he accidentally gave her \$50.27 instead.) After buying a newspaper for 50 cents, Ms Smith noticed that she had left exactly three times as much as the amount on the original check. What was the amount of the check?

    - If the amount of the check is \$Y.X where Y and X are each numbers between 0 and 99, then the problem tells us that $100Y + X - 50 = 3(100X + Y)$, which reduces to $50 = 97X - 299Y$. This equation has infinitely many solutions over reals (indeed, over integers) but only one in integers for which X,Y are between 0 and 99–namely, $X = 56$, $Y = 18$. (Note, this is not easy to figure out!) So the amount of the check was \$18.56.