Geometry Homework Week 2

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Throughout this handout the notation (ABC) refers to the circumcircle of $\triangle ABC$.

1 Directed Angles Exercises

1. (Isosceles triangle) $AB = AC$ if and only if $\measuredangle ACB = \measuredangle CBA = -\measuredangle ABC$ (not $\measuredangle ACB = \measuredangle ABC$!)
2. (Not isosceles triangle) If A does not lie on $\overline{BC}$ then $\measuredangle ACB = \measuredangle ABC$ implies $B = C$.
3. (Inscribed angle theorem) If (ABC) has center P then $\measuredangle APB = 2\measuredangle ACB$
4. (Parallel Lines) If $\overline{AB}||\overline{CD}$ then $\measuredangle ABC + \measuredangle BCD = 0$.
5. (Cyclic quadrilaterals) Points A, B, X, Y lie on a circle if and only if $\measuredangle AXB = \measuredangle AYB$.

2 Problems

1. (Incenter/Excenter Lemma AKA Fact 5) Let I be the incenter and I_A be the A-excenter of triangle ABC. Let L be the midpoint of arc $\widehat{BC}$ (not containing A) of the circumcircle of ABC. Prove that L is the center of a circle passing through B, I, C, I_A.
2. (EGMO Lemma 1.42) Let ABC be an acute triangle inscribed in circle Ω. Let X be the midpoint of the arc $\widehat{BC}$ not containing A and define Y, Z similarly opposite B, C. Show that the orthocenter of XYZ is the incenter I of ABC.
3. (Example 1.32) Let ABC be an acute triangle with circumcenter O, and let K be a point so that $\overline{KA}$ is tangent to (ABC) and $\angle KCB = 90$. Point D lies on BC so that $\overline{KD}||\overline{AB}$. Show that line $\overline{DO}$ passes through A.
4. (Lemma 1.44) Let ABC be an acute triangle. Let $\overline{BE}$ and $\overline{CF}$ be altitudes of ABC, and denote by M the midpoint of $\overline{BC}$. Prove that $\overline{ME}, \overline{MF}$, and the line through A parallel to $\overline{BC}$ are all tangents to (AEF).