

ORMC Week 2: Algebra

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Algebraic Simplification

$$(a+b)^2 \neq a^2 + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$(a+b)(a^2 - ab + b^2) = a^3 + b^3$$

$$(a-b)(a^2 + ab + b^2) = a^3 - b^3$$

Algebraic Manipulation Example

If the sum of 2 numbers x and y is 12 and the sum of the squares is 20, find the product of the 2 numbers.

Functions

A function is an operation that relates an input value, usually x , to an output value. Every input must have only one output.

The input is the **domain** and the output is the **range**.

Functions are usually denoted as $f(x)$, although these variables can be replaced with anything.

Practice:

- What is the domain and range of $f(x) = (4-x)/(12+x)$?
- If $f(x) = 2x+3$, find $f(1) + f(0)$.

Function Rules

$$f(x) + g(x) = (f+g)(x)$$

$$f(x) - g(x) = (f-g)(x)$$

$$f(x)g(x) = (f \cdot g)(x)$$

$$f(x)/g(x) = (f/g)(x)$$

Composition of functions: $f(g(x)) = (f \circ g)(x)$

Inverse function: If $f(x) = y$, then $f^{-1}(y) = x$.

Practice:

- Find the inverse function of $f(x) = 14x-7$.

Recursive vs. Explicit Functions

An explicit function gives you the value of $f(x)$ for any x .

Ex.) $f(x) = 17x - 1$

A recursive function gives you the value of $f(x)$ using the previous value(s).

Ex.) $f(x) = f(x-1) + f(x-2)$ where $f(0) = 0$ and $f(1) = 1$

- Find $f(5)$ of the above recursive function.

Exponents & Radicals

When you raise a number to an exponent, you multiply the number to itself that many times.

$$2^3 = 2 * 2 * 2 \neq 2 * 3$$

A radical is a fractional exponent.

Practice:

- $32^{2/5} =$

- $27^{4/3} =$

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

$$\sqrt[n]{x^y} = x^{\frac{y}{n}}$$

$$\sqrt{2} = 2^{\frac{1}{2}}$$

$$\sqrt[3]{4} = 4^{1/3}$$

$$\sqrt[5]{2^3} = 2^{\frac{3}{5}}$$

Logarithms

A logarithm is the inverse function of an exponent.

If $a^x = b$, then $\log_a x = b$.

Logarithms solve for the exponent.

If there is no base, then we assume it is 10. $\rightarrow \log 2 = \log_{10} 2$

Natural log has base e. $\rightarrow \ln 2 = \log_e 2$

Practice:

- $\log_2 1024 =$

Exponent & Logarithm Rules

$$x^m \cdot x^n = x^{m+n}$$

$$x^m / x^n = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$x^0 = 1$$

$$\log(xy) = \log(x) + \log(y)$$

$$\log(x/y) = \log(x) - \log(y)$$

$$\log(x^y) = y\log(x)$$

$$\log 1 = 0$$

$$\log_a b = \log(b) / \log(a) = \ln(b) / \ln(a)$$

^ work for any base!

Algebraic Manipulation Questions

1. If $3x + \frac{1}{2x} = 3$, what is the value of $8x^3 + \frac{1}{27x^3}$?
2. The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?
3. If a and b are real numbers such that $a + b = 7$ and $ab = 5$, what is $a^3 + b^3$?

Functions Practice Questions

1. Let f be a function such that $f(x-y) = f(x) + y$ for all real numbers x and y and $f(40) = 20$. What is $f(30)$?
2. A function is defined recursively by $f(1) = f(2) = 1$ and $f(n) = f(n-1) - f(n-2) + n$ for all integers $n \geq 3$. What is $f(2018)$? (2018 AMC 10B)
3. Let $h(4x-1) = 2x+7$. For what value of y is $h(y) = y$?

Exponents/Logarithms Questions

1. Positive real numbers $x \neq 1$ and $y \neq 1$ satisfy $\log_2 x = \log_y 16$ and $xy = 64$. What is $(\log_2 \frac{x}{y})^2$?
2. What is the value of $(625^{\log_5 2015})^{\frac{1}{4}}$?
3. Which of the following is the value of $\sqrt{\log_2 6 + \log_3 6}$?
(A) 1 (B) $\sqrt{\log_5 6}$ (C) 2 (D) $\sqrt{\log_2 3} + \sqrt{\log_3 2}$ (E) $\sqrt{\log_2 6} + \sqrt{\log_3 6}$