

## Logic Puzzles: Wizards and Hats

UCLA Olga Radko Math Circle 10/18/2020

This week, we will be exploring some logic puzzles.

### Warm Up:

In the past, we visited the Island of Knights and Liars to solve some tricky puzzles. Let's revisit one of the puzzles we saw.

Remember, **Knights** always tells the truth and **Liars** always lie.

1. *Charlie, Donna, and Emily are islanders, not tourists. A tourist asked Charlie if he was a knight. Charlie mumbled something, chewing his gum, so the tourist did not understand his answer. The tourist asked Donna what Charlie said. Donna answered that Charlie claimed to be a liar. To that Emily responded, "Don't believe Donna, she is a liar herself!" What kinds are Charlie, Donna and Emily?*

Charlie: \_\_\_\_\_

Donna is a \_\_\_\_\_

Emily is a \_\_\_\_\_

We also learned about a very evil king who had a very unique way of punishing criminals. Law-breakers were given a choice between two doors. Behind each door, there was either a **hungry tiger** or a **treasure of gold**, but **not nothing** or **both**. The king would also post warnings on the doors and let the criminals choose.

2. *One day, there was a criminal facing the doors with the following signs.*

Door 1: *at least one of these rooms has gold*

Door 2: *a tiger is in the other room*

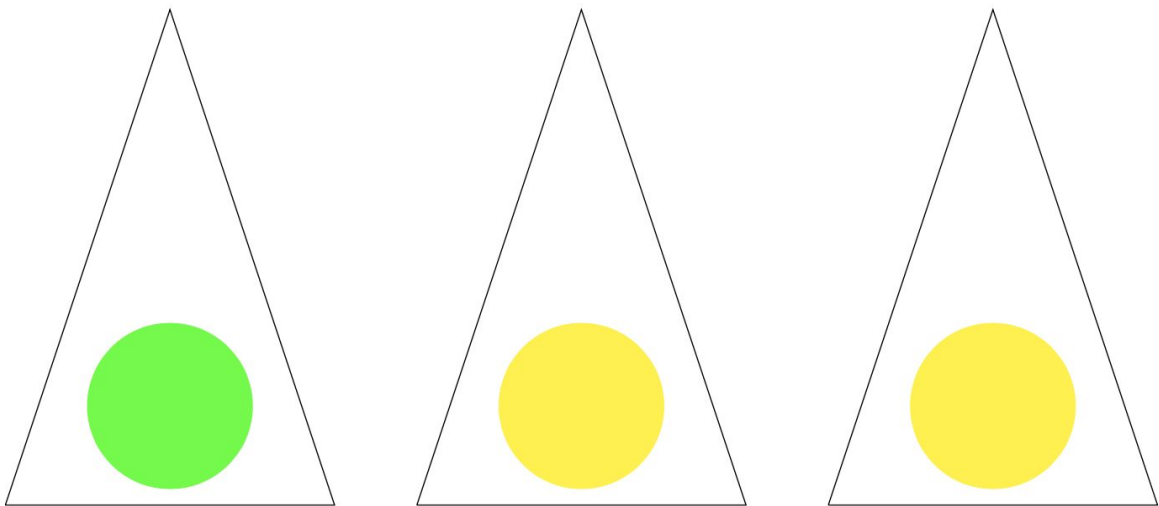
*"Are the signs true?" asked the prisoner. "They are either both true or both false," replied the king. Then he smiled warmly and added, "Make your choice, buddy!"*

*Which door should the prisoner open? Why?*

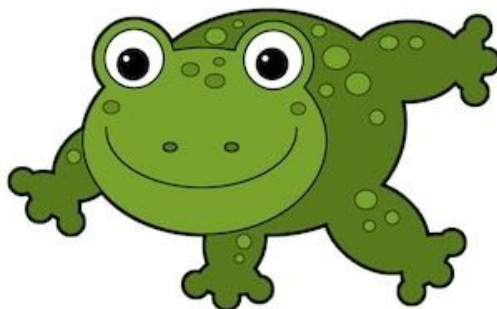
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**Lesson:**

A part of wizardry training is developing logic skills. In the following exercises, two students will be presented with three wizard's hats, one with a green label and two with yellow labels.



The students will be asked to close their eyes. One hat will be hidden, the others will be put on the students' heads. The students will open their eyes, and then be asked, "Who knows what color hat he/she has on? If so, which color is it?" The students can figure out the color hat he/she is wearing only by observing the hat of the other student. **A student is not allowed to look at his/her own hat.** This kind of cheating will be punished by turning the cheat into a toad!



**Problem 1:** Kaley and Emmanuelle are given two hats. Kaley gets the green- labeled hat, and Emmanuelle gets a yellow-labeled one.

*a) Cory, the lead wizard, asks them what color hat they have on. What are Kaley and Emmanuelle going to say? Explain.*

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

*b) After hearing Emmanuelle's answer, can Kaley now figure out which color hat she has on? Explain why or why not.*

**Problem 2:** This time, Kaley and Emmanuelle are both given the yellow- labeled hats.

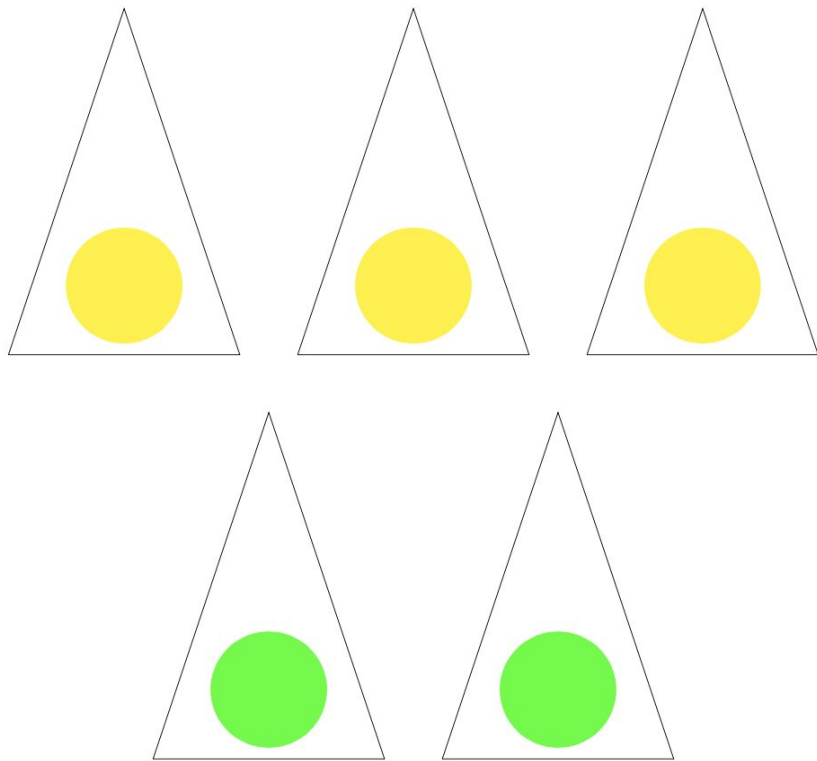
*a) Cory asks them what color hat they have on. What are Kaley and Emmanuelle going to say? Explain.*

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

*b) Now that they heard each other's answers, can either of them figure out the color of their hat? How?*

As the training progresses, three students will be chosen and presented with five hats, three with yellow dots and two with green. Once again, the students will be asked to close their eyes. Two hats will be hidden, three will be put on the students' heads. The students will be asked to open their eyes and to figure out the color of their hat labels by observing those of the other students.



**Problem 3:** Kaley, Emmanuelle, and Lucy are given the hats. Kaley gets a yellow-labeled hat, Emmanuelle and Lucy get the hats with green labels.

a) *What will each person say when Cory asks them for the color of their hats? Explain.*

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

Lucy: \_\_\_\_\_

b) *After hearing each person's answer, can others figure out which color hat they have on? How?*

**Problem 4:** This time, Kaley and Emmanuelle get yellow-labeled hats while Lucy gets the hat with a green label.

a) *How will each person respond to Cory's question? Explain.*

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

Lucy: \_\_\_\_\_

b) *Now that they have heard each other's answers, who can say the color of their hat and who can't? Explain why or why not. (Hint: What would happen if one assumes that she is wearing the green hat?)*

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

Lucy: \_\_\_\_\_

c) *After hearing everybody's answers, who can now figure out the color of their hat? Explain. (Hint: What would happen if Lucy assumes she is wearing the yellow hat?)*

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

Lucy: \_\_\_\_\_

**Problem 5:** Finally, all the three students are given the yellow-labeled hats.

a) *How would each person respond to Cory's question? Explain.*

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

Lucy: \_\_\_\_\_

b) After hearing everybody's answer, who can say the color of their hat and who can't? Explain why or why not. (Hint: What would happen if one of them assumes she is wearing a green hat?)

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

Lucy: \_\_\_\_\_

c) After hearing each person's answer, Lucy tries to figure out the color of her hat. **She initially makes the assumption that she is wearing a green hat.**

i) If Lucy was wearing the green hat, would anybody have immediately known which color hat they had on?

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

Lucy: \_\_\_\_\_

ii) Does this agree with what you wrote for part a? \_\_\_\_\_

iii) After hearing everybody's answer, would anybody be able to figure out the color of their hat? (Remember, Lucy is assuming that she has a green hat on.)

Kaley: \_\_\_\_\_

Emmanuelle: \_\_\_\_\_

Lucy: \_\_\_\_\_

*iv) Does this agree with what you wrote for part b? \_\_\_\_\_*

*v) If the answer is not the same as part b, explain what this contradiction means.*

*vi) Does Lucy know which color hat she has on?*

*vii) Does everybody else know?*



### Challenge Problem:

We have an infinite supply of yellow and green hats. The students sit in a line and each student can see all of the hats in front of them, but not their own or those behind them. The instructor starts at the back of the line and one by one asks each student which color hat they have. **The students know how the instructor is going to ask, and hence devise a plan to maximize the total number of correct guesses.**

For each number of students, please explain the students' plan and how many hats they are certain to guess correctly.

1) *There are 2 students and Student 1 is sitting behind Student 2. Since Student 1 is at the back of the line, the instructor asks Student 1, "What color hat do you have on?" Afterwards, proceeds to Student 2 and asks the same question.*

a) *Is it possible for both Student 1 and Student 2 to correctly guess the color of their hats on the first try?*

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b) *If it is not possible for both students to guess correctly for the first time, is there a strategy the two students can develop such that **at least one student** guesses their hat correctly?*

c) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

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2) *There are 3 students. Student 3 sits in front of Student 2 and Student 1 is at the back of the line.*

a) *Is it possible to expand the strategy developed in the 2-student scenario for the three students? If not, try to revise the strategy.*

b) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

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3) *There are 4 students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

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4) *There are 5 students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

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5) *There are 6 students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

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6) *There are 7 students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

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*Are you starting to see a pattern?*

7) *There are  $n$  students.*

- a) *What is the maximum number of correct guesses the students can **always** achieve **in certainty**?*

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