

Geometry Homework Week 1

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- (Incenter/Excenter Lemma AKA Fact 5) Let I be the incenter and I_A be the A -excenter of triangle ABC . Let L be the midpoint of arc \widehat{BC} (not containing A) of the circumcircle of ABC . Prove that L is the center of a circle passing through B, I, C, I_A .
The A -excenter I_A is the center of the circle which is tangent to segment BC and lines (not segments) AB and AC with center on the opposite side of BC from A .
- (EGMO Lemma 1.42) Let ABC be an acute triangle inscribed in circle Ω . Let X be the midpoint of the arc \widehat{BC} not containing A and define Y, Z similarly opposite B, C . Show that the orthocenter of XYZ is the incenter I of ABC .
- (USAJMO 2011/5) Points A, B, C, D, E lie on circle ω and point P lies outside the circle. The given points are such that (i) lines PB and PD are tangent to ω , (ii) P, A, C are collinear, and (iii) $\overline{DE} \parallel \overline{AC}$. Prove that \overline{BE} bisects \overline{AC} .
- (EGMO Lemma 1.45) The incircle of ABC is tangent to BC, CA, AB at D, E, F respectively. Let M and N be the midpoints of BC and AC . Ray BI meets line EF at K . Prove that K, M, N are collinear. Hint: Phantom Point.