

**Pooled Testing**  
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**INTRODUCTION:**

The Fake Coin Problem

Out of all the Math Circle worksheets I have ever done, my favorite is the worksheet about fake coins. That worksheet asks questions like these.



1. Suppose you have 3 coins. One of them is fake and is lighter than the other two. Use a balance to find the fake coin. How many trials do you need? Can you find the fake coin in just one trial?

2. You now have 9 coins. One of them is fake. The fake coin is lighter than the real coins. Can you always find the fake coin? Can you do it in just 2 trials?

## **APPLICATION:**

### Pooled Testing

The fake coin problems always seemed to be pure mathematical fun, but with little real application to my life. I was never going to be a gold coin detective, after all. But then the pandemic hit, and I saw the fake coin problem arise anew. How? Consider the challenge of reopening the UCLA school campus.

1. UCLA would like to open its physical campus but is worried about the spread of coronavirus. The administration has therefore decided to test every student, every day, in the hope of quickly detecting any possible source of exposure. The test works as follows. A student spits in a cup. That student's saliva is then exposed to a chemical that turns the sample red if there is any sign of infection, but green otherwise. Suppose that the test is 100% accurate.

If UCLA needs to test 100 students, the obvious way to do so is to collect 100 saliva samples and test each of them separately. Imagine that 1 student is infected, and 99 are not. If you test each student individually, how many green samples would you get? How many red? How many tests would you run?

2. Now start again but use the lessons of the fake coin problem. Ask 50 students to all spit into one container, and ask the other 50 students all spit into another container. Run two tests. How many green samples do you expect? How many red samples?

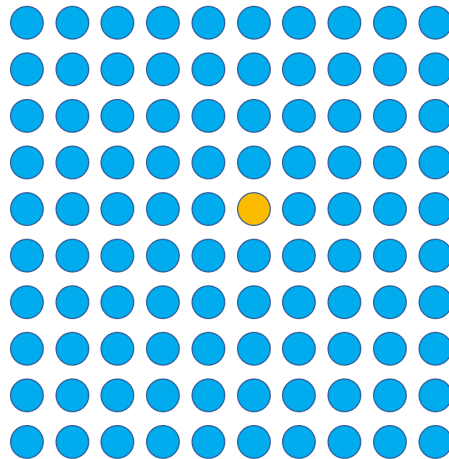
3. Focus now on the students who spat into the sample that turned red. What would you do next? And after that? What is the smallest number of tests you would need to run in order to find the one infected student with 100% confidence?

## THE DIAGRAM

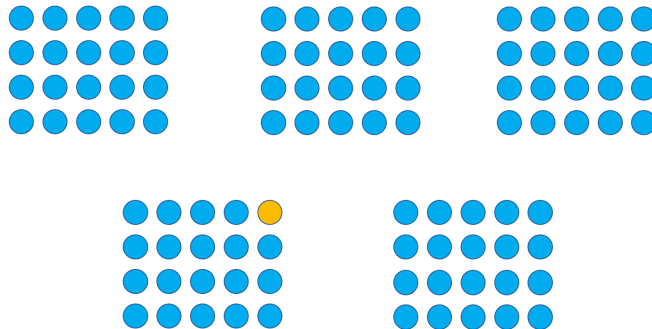
### Pooled Testing in Action

We can draw a quick sketch to show how pooled testing can help us reduce the number of tests we need. Remember, we are comparing this method to the obvious alternative of testing all 100 kids separately. That method would clearly work, but it might be costly and/or slow.

1. Start with 100 students. The orange circle represents the one sick person.



2. Randomly divide the students into five groups of 20. Run five tests. How many turn red? How many turn green?



Focus on the one sample that turned red. Take those 20 students and break them into five groups of four. Run four tests. How many will turn red? How many green?



3. Focus on the one sample that turns red. Test each of those five students separately. How many total tests have you run? Have you found the one sick student?

4. Assume that only 1 student is sick out of the 100 students in our class. Also assume that our test is 100% accurate. What pattern of testing uses the smallest number of tests to find the one sick student with confidence? Should you start with two groups of 50? Five groups of 20? Ten groups of 10? Something else? Why?

## THE REAL WORLD

What if the infection rate is high?

1. So far, we have assumed that only 1 student in every 100 students will be sick. But what if the infection rate were to increase? Would that change anything about our thinking? For example, what if 2 students out of every 100 are sick. Would you test in a different pattern?

2. What about if 10 students out of every 100 are sick?

3. Will pooled testing always use fewer tests than individualized testing, no matter the underlying rate of infection in the community? Explain.

## THE REAL WORLD

What if the test has false positives?

1. So far, we have assumed that the saliva test is perfect. But what if it has some false positives. That is, what if the solution will sometimes turn red even when the relevant students are all in fact healthy? How would that impact the standard strategy of testing each student separately?

2. Now, how would that change the pooled testing strategy? Would it lead you to change your pattern of testing? Would it make pooled testing less attractive? More attractive? Explain.

## THE REAL WORLD

What if the test has false negatives?

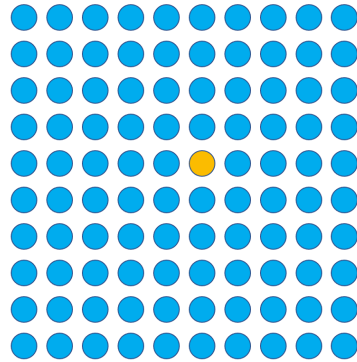
1. Now try the opposite assumption. Imagine that the solution sometimes turns green even when the relevant students are all in fact infected? How would that impact the standard strategy of testing each student separately?

2. How would that change the pooled testing strategy? Would it lead you to change your pattern of testing? Would it make pooled testing less attractive? More attractive? Explain.

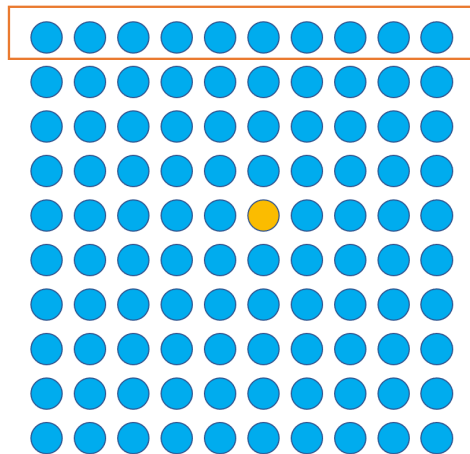
**EXTENSION**

EACH SAMPLE INCLUDED IN TWO GROUPS

1. Start again with 100 students, where only one is sick.

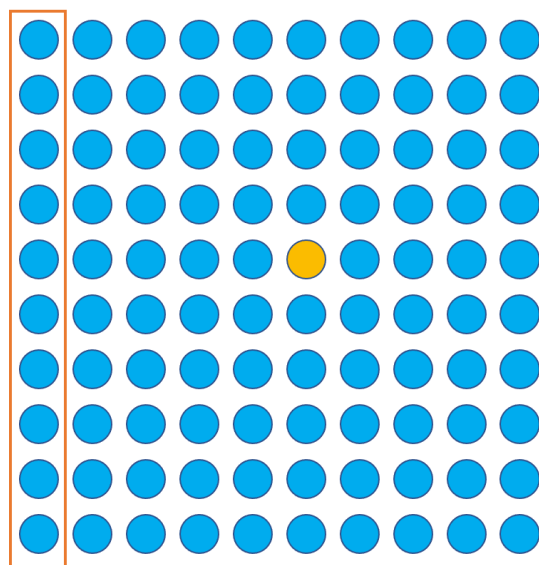


2. Imagine creating 10 pools, one for each row in the chart. As an example, the first row is shown in an orange rectangle. There would be nine others.





3. Imagine also creating 10 pools based on the columns. As an example, the first column is shown in an orange rectangle. There would be nine others.



4. Out of the 20 tests described above (10 pooled rows, 10 pooled columns), how many samples would turn red? How many green?

5. How many total tests would it take in order to find the one sick student using this grid-like testing approach? When might this approach be better than the pooled testing approach introduced at the start of this handout? When worse?

### BONUS

#### What now?

Seeing all these complexities, how would you go about deciding your testing strategy? Would you hire a math PhD and ask them to run all the numbers? If not, what would you do? (**Hint:** A version of this worksheet was originally created for use in our computer coding club. Do you see why?)