

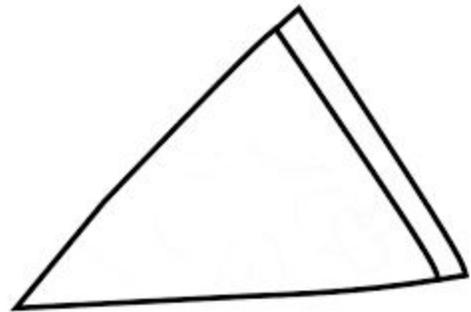
# “I Cut You Choose”

Olga Radko Math Circle Beginners 2

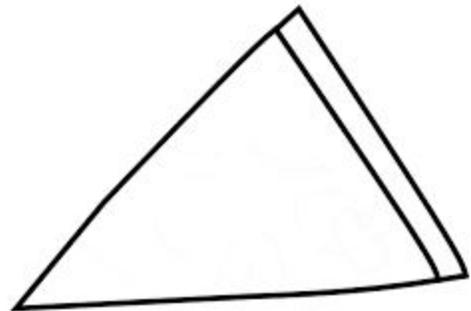
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**Warm-up:** Here is a slice of pizza. My best friend and I must share this slice. Feel free to add any additional details on the pizza you would like.

a) How am I going to split the slice? Why?

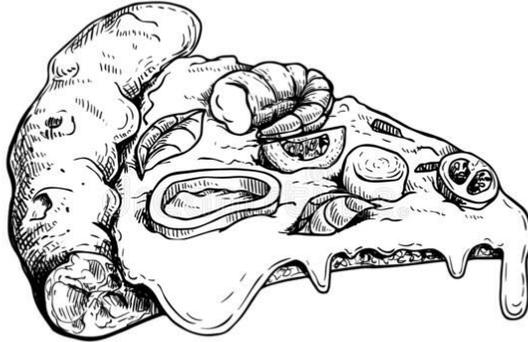


b) How would your best friend split? Why?



c) Who is happier in each situation, and why?

**Setting:** I have one younger sister and her name is Ashly. Growing up, my sister and I got along pretty well, except when it came to seafood pizza. That's because my sister and I both love seafood pizza to the point where we would often fight, whine, and cry over it, especially if we feel that the other has the better slice.



Questions (Please provide your reasoning for each):

1. I am greedy and selfish. Cut the pizza randomly into two uneven pieces.



- a. However, if my sister gets to pick first, which of the two pieces will she pick?
- b. Which of the two pieces am I going to be left with?
- c. Who is happy here and who is sad?

2. What happens if we value the slice of pizza differently?
- a. What if my sister is only 3 years old and has a very small appetite?



- b. What if my sister hates shrimp, and I LOVE shrimp?



- c. What if I hate the crust of pizza?



3. In Question 1, I cut the slice of pizza without knowing that my sister would get to choose first, leaving me very sad and angry. So does the order of who does the cutting and who does the choosing matter?

a. How would I divide the pizza when I cut and choose first?



b. How would I divide the pizza when I cut first, but my sister chooses first?



c. How would my sister divide the pizza if she cuts and chooses first?



d. And if she cuts, but I choose first?



4. How will I divide a slice of pizza so that we are both equally happy?

a. Does it need to be exactly symmetrical?

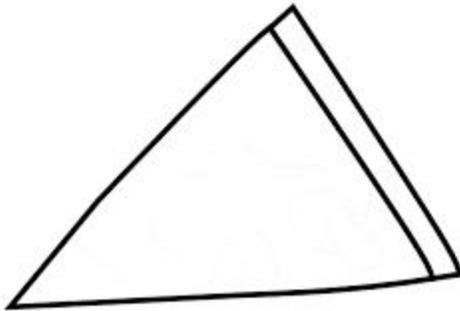
b. When does it need to be symmetrical?

c. Can you divide the slice equally by cutting it more than just one time?

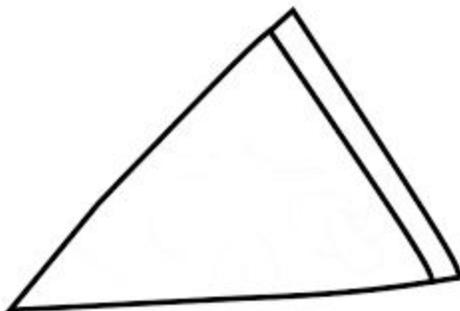
So notice that how I divide the slice of pizza depends on a number of factors. In order for the cutting to be “fair”, we need to consider how to make it envy-free so that both individuals are equally happy with their portion. This includes taking into account the subjective value (how each individual values the pizza). I might love pizza, hate pizza, or only like the crust or certain toppings!

5. Does dividing the slice of pizza to be fair imply that it is efficient? In other words, can we always get the maximum value (summing all positive and negative feelings) when we try to make it fair? Take a guess first and explain.

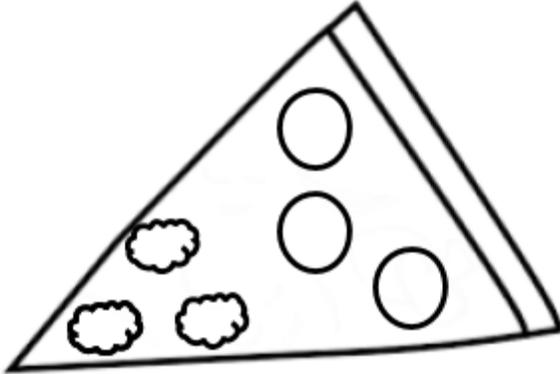
a. Can you come up with an example that is efficient and envy-free?



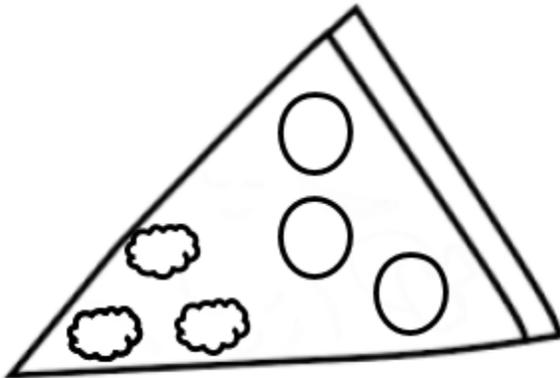
b. Can you come up with an example that is NOT efficient, but still envy-free?



- c. If you only love pepperoni and hate sausages, but your sister hates pepperoni and only loves sausages, how would you divide the slice to be envy-free?



- d. Is this the most effective way to cut it? Is there a way to make the slices envy-free, but you and your sister even more satisfied than before?



**Conclusion:** What we have discussed is the basics of envy-free cake-cutting, but using an example with pizza instead of cake. Our goal is to find a way to divide the slice of pizza such that nobody feels envious over another person's share. In other words, there is no jealousy as each individual feels equally satisfied with everyone's share. We then also thought about the relationship between being envy-free and being efficient. This packet only includes examples between two agents, but feel free to ponder how to divide an object to be envy-free with 3 or more agents.