Introduction to Number Theory (Reading)

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9 August 2020

1 Clocks and Periodicity

Time, like time on an actual clock with minutes and hours, is not discussed from an absolute starting point. For example, there is no 42,724,601 hour. We "restart time" on the clock periodically and return to 12 o'clock position, for 12 hour time, after the 11^{th} hour has passed. There is some sense of *periodicity*, or periodic returning, of the actual numerical value in time. For example, we know that 52 hours from 2PM is 6PM in time. Why? Well, $52 = 48 + 4 = 2(24) + 4$, so two whole 24 hour periods, and a leftover of 4 hours after 2PM is 6PM. We have a remainder of 4 hours after 2PM giving us 6PM.

Figure 1: Diagram of an analog clock, numbers on the clock are periodic.

Consider this: look at your watch, at the time, and realize that we can start counting from the time that you are looking at and *define* this as the start of your *universe*. Then 24 hours from the start of your *universe* would be time 24. Another 24 hours would be time 48; another 24 would be time 72; and so on. But using the definition of periodic time, and assuming that there are 24 hours in your *day*, the actual values of times 24, 48, 72, and so on, are all the same time if we define a period.

For example, let our period be 24. Then time 24 is equal to time 48, is equal to time 72, and so on. More accurately, we would say that 24 is congruent to 48 mod 24, 72 is congruent 48 mod 24... Just like in geometry there are "congruent" triangles that differ only by some multiples of side lengths, but whose dimensions are essentially the same. We explore this idea in more detail in our class and its properties, as well as extensions and results of this definition.

But what if we are interested in other numbers, number systems, etc... What if we were on another planet, say Venus? Well, days on Venus last 116 Earth days and 18 hours. The clocks on Venus, to tell the time in the day for the Venetian, could probably be on much longer periods...

Here's another problem to consider: How many counting numbers are there? Is there a way for us to *represent* and *capture* as many counting numbers as possible, say on a computer, without actually storing each and every digit?

In this way, talking about different times, talking about different number systems, is useful. We can *characterize* potentially very large numbers and pack them into multiples of other numbers and their remainders. Instead of 42,724,601 hours since the world arbitrarily decided to start counting time, we can just say that in a day there are 24 hours in a day, and at least for 12 hour time, we periodically return every 12 hours to same numerical value of time on the clock.