

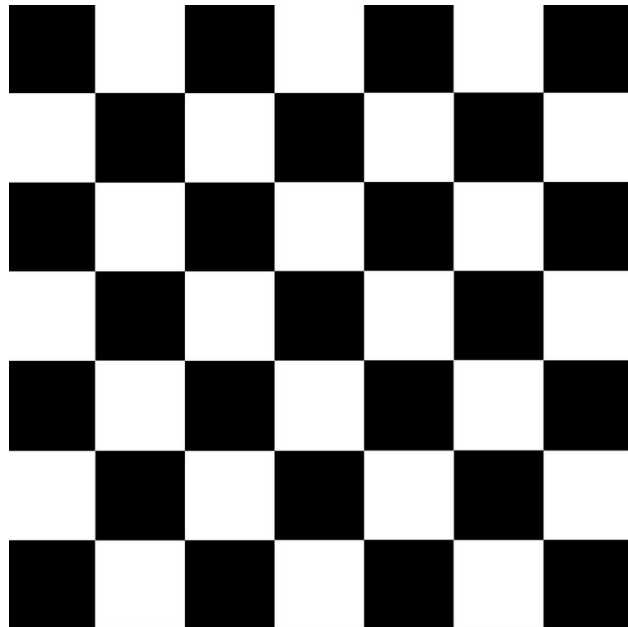
Graph Theory II

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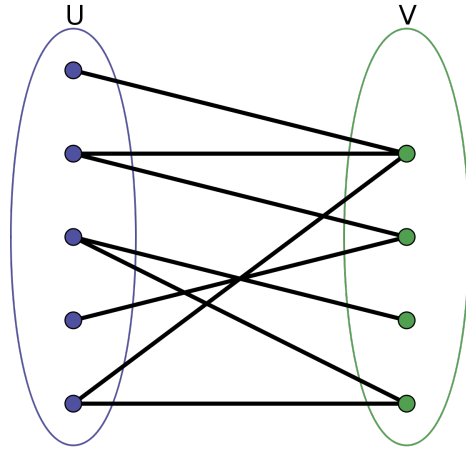
Problem 1 An Eulerian path in a graph is a path which passes through every edge exactly once. Show that there exists an Eulerian path in a given graph if and only if the graph is connected and all but exactly two vertices have even degrees.

Problem 2



Is it possible to walk around the 7×7 chess board with a knight visiting every square exactly once and finishing back at the starting square?

Problem 3



1. A bipartite graph has b white and r black vertices. What is the maximum possible number of edges in this graph?
2. What is the maximum possible number of edges in a bipartite graph with $2n$ vertices?
3. What about $2n + 1$ vertices?

Problem 4

Show that a graph where every vertex has degree 2 is a collection of disjoint simple cycles.

Problem 5

On a test every student solved exactly 2 problems, and every problem was solved by exactly 2 students.

1. Show that the number of students in the class and the number of problems on the test are the same.
2. The teacher wants to make every student present one problem they solved at the board. Show that it is possible to choose the problem each student presents so that every problem on the test gets presented exactly once.