

Combinatorics

Overview:

- I. What is Combinatorics?
 - II. Factorials
 - III. Choose Function
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I. What is Combinatorics

Combinatorics → mathematical study of counting
Algebra → mathematical study of relationships
(Calculus → mathematical of change, area)

II. Factorials

Factorials answer the question “how many ways are there to arrange n objects?” We define this quantity to be $n!$, and it can be calculated as follows.

$$n! = n(n - 1)(n - 2) \dots 1$$

Some key values you should memorize are

$$\begin{aligned}5! &= 120 \\4! &= 24 \\3! &= 6 \\2! &= 2 \\1! &= 1 \\0! &= 1\end{aligned}$$

III. Choose Function

A key application of factorials is the choose function. The choose function helps us answer the question “how many ways can one choose k objects out of n objects?” This question is subtle, and the answer depends on if the order in which one choose the objects matters.

If order doesn't matter, it's a **combination**

$$\binom{n}{k} = C_k^n = \frac{n!}{(n - k)! k!}$$

If order does matter, it's a **permutation**

$$P_k^n = \frac{n!}{(n - k)!}$$