Combinatorics

Overview:

I. What is Combinatorics?
II. Factorials
III. Choose Function

I. What is Combinatorics

Combinatorics \( \rightarrow \) mathematical study of counting
Algebra \( \rightarrow \) mathematical study of relationships
(Calculus \( \rightarrow \) mathematical of change, area)

II. Factorials

Factorials answer the question “how many ways are there to arrange \( n \) objects?” We define this quantity to be \( n! \), and it can be calculated as follows.

\[
n! = n(n - 1)(n - 2) \ldots 1
\]

Some key values you should memorize are

\[
\begin{align*}
5! &= 120 \\
4! &= 24 \\
3! &= 6 \\
2! &= 2 \\
1! &= 1 \\
0! &= 1
\end{align*}
\]

III. Choose Function

A key application of factorials is the choose function. The choose function helps us answer the question “how many ways can one choose \( k \) objects out of \( n \) objects?” This question is subtle, and the answer depends on if the order in which one choose the objects matters.

If order doesn’t matter, it’s a combination

\[
\binom{n}{k} = \binom{n}{k} = \frac{n!}{(n - k)! \cdot k!}
\]

If order does matter, it’s a permutation

\[
\begin{align*}
\sigma_k^n &= \frac{n!}{(n - k)!} \\
\end{align*}
\]