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Intro to geometry, or is your world flat?

Part 1

Here are some standard facts from geometry of the Euclidean plane.

1. For any two distinct points in the plane, there exists a unique straight line passing through them.

2. The shortest path connecting two distinct points of the plane is the segment of the straight line passing through them.

3. For any straight line and a point away from it, there exists a unique straight line passing through the point parallel to the original line (the modern version of Euclids fifth postulate, possibly the most controversial scientific statement of all times).

4. The simplest possible polygon in the Euclidean plane is a triangle.

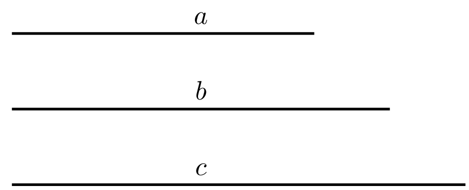
5. For any three points in the plane not belonging to a straight line, there exists a unique triangle having these points as vertices.

6. The sum of the angles of any triangle in the plane equals π.

7. For any point of the plane and any positive real number, there exists a unique circle centered at the point and with the radius equal in length to the number. The circle has one connected component (cannot be represented as the union of two or more disjoint nonempty open subsets).

First, we shall familiarize ourselves with some of the above facts.

Problem 1 *Using a compass and a ruler, draw a triangle with the given sides* a*,* b*, and* c*.*

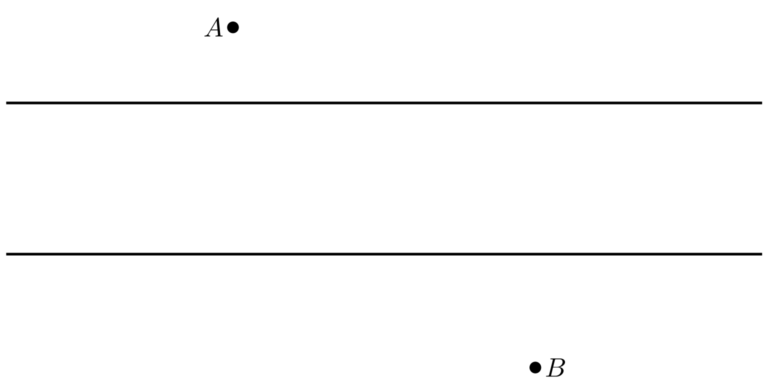


Problem 2 *Prove that the sum of the angles of any triangle in the Euclidean plane equals* π*.*

Problem 3 *Prove that midpoints of the sides of any quadrilateral in the Euclidean plane are vertices of a parallelogram. For what quads is the parallelogram*
a. a rhombus, b. a rectangle, c. a square?

Problem 4 *Given a straight line and a point away from it, use a compass and a ruler to draw the straight line passing through the point parallel to the original line.*

Problem 5 *The cities* A *and* B *are separated by a river having straight banks.*



You need to build a highway connecting A to B. The bridge across the river must be orthogonal to the banks. Construct the highway as short as possible.

Now it's time to study the first 2D surface different from the Euclidean plane, a cylinder. We shall need some sticky tape and scissors for the following.

Problem 6 *Take a sheet of paper. Imagine that it is glued into a cylinder. Take two arbitrary points on the surface. Draw the shortest path connecting the points. Glue the sheet into a cylinder to see what you get.*

Problem 7 *Imagine that your paper cylinder is a part of an infinite cylinder. Extend the shortest path from the previous problem to a "straight line" (a helix) on the latter.*

Problem 8 *Take three generic points on a cylinder. How many triangles having the points as vertices can you draw? How many triangles without self-intersection can you draw?*

Problem 9 *What is the sum of the angles of a triangle on a cylinder?*

Problem 10 *How many "straight lines" (geodesics) passing through two distinct points can you draw on a cylinder?*

Similar to the Euclidean plane, let us define a circle as the set of all the points of the surface having a specified distance, called the radius, from a special point, called the center.

Problem 11 *Draw a circle on a cylinder*

1. with the radius shorter than the half-length of the cylinder's circular "waist";

2. with the radius longer than a half of the "waist".

How many connected components do you get in each case?

Undertake a similar (starting with Problem 6) exploration of the Möbius surface.