Math Circles Intermediate 2A - Induction 1

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July 12, 2020

Problem 1 [1]

a. Show that

$$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

b. Show that

$$1 + 3 + \ldots + (2n-1) = n^2$$

c. Show that

$$2 + 5 + \ldots + (3n-1) = \frac{3n^2 + n}{2}$$

d. Show that

$$1 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Problem 2

Show that the number $111\ldots111$, consisting of 243 ones, is divisible by 243. Prove the generalization: Show that for any positive integer n, the number consisting of 3^n ones is divisible by 3^n.

Problem 3

Show that $n^3 + 2n$ is divisible by 3 for all positive integers n.
hint: Think about what it means for a number to be divisible by another number. Formulate this mathematically. Also, when you get to the inductive step, expand $(k+1)^3 + 2(k+1)$ and you should notice some things ...

Problem 4

a. Show that for any positive integer n we have $2^n > n$.
b. Find all positive integers n such that $2^n > n^2$ and prove your result.

Problem 5

Suppose there are n lines drawn in the plane such that no two lines are parallel and no three lines intersect at the same point. Find a closed formula for the number of regions that the lines split the plane into.

[1] Problems from this handout were compiled from https://circles.math.ucla.edu/circles/lib/data/Handout-2293-2048.pdf and https://circles.math.ucla.edu/circles/lib/data/Handout-2290-2046.pdf