

Sticks and Stones (part 2)
Version 1.0

5/31/20

Start with the homework problems.

1. The players on my sports team scored 20 points in total in today's game. I have three players. How many different scoring patterns possibly happened? Assume that every player scored at least 1 point.

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19 spots
2 sticks

$$\frac{19 \cdot 18}{2} = 19 \times 9 = 171$$

2. Same question, but now consider the possibility that a player might score zero points. For example, it might be that two of my players both scored zero points, and all points were therefore scored by my star third player. Now how many possibilities are there?

$$A+B+C = 20$$

↓

$$A+B+C = 23$$

22 spots
2 sticks

$$\frac{22 \times 21}{2} = 231$$

OR

$$\frac{20+2}{20! 2!}$$

3. HARDER. I roll three normal six-sided dice. If the three rolled numbers add up to 10, I win. Thus, a roll of 1, 3, and 6 would win. But a roll of 4, 4, and 5 would not. How many different ways can I win? (Hint: If your answer is 36, you are close but not quite there. Think about the difference between this problem and a problem like "I have three numbers, and they add up to 10.")

$$A+B+C = 10 \text{ but eliminate } 7, 8, 9, 10 \dots \text{ and } 0$$

9 spots
2 sticks

$$\frac{9 \times 8}{2} = 36$$

$$\begin{array}{r} 36 \\ - 9 \\ \hline 27 \end{array}$$

10 won't happen

9 won't happen

$$\begin{array}{l} 8 + 1 + 1 \\ 1 \ 8 \ 1 \\ 1 \ 1 \ 8 \end{array} \left. \vphantom{\begin{array}{l} 8 + 1 + 1 \\ 1 \ 8 \ 1 \\ 1 \ 1 \ 8 \end{array}} \right\} \leftarrow \text{No good}$$

← 9

$$\begin{array}{l} 7 + 2 + 1 \\ 7 \ 1 \ 2 \\ 2 \ 1 \ 7 \\ 2 \ 7 \ 1 \\ 1 \ 7 \ 2 \\ 1 \ 2 \ 7 \end{array} \left. \vphantom{\begin{array}{l} 7 + 2 + 1 \\ 7 \ 1 \ 2 \\ 2 \ 1 \ 7 \\ 2 \ 7 \ 1 \\ 1 \ 7 \ 2 \\ 1 \ 2 \ 7 \end{array}} \right\} \leftarrow \text{No good}$$

9. Another way to think about $A+B+C = 10$ with zeros allowed.

10 stones, 2 sticks, $\frac{12!}{10! 2!} = \frac{12 \cdot 11}{2}$ ✓

10. The telephone number problem. Non-decreasing 9-0, 8 digits

WAIT

11. When 7 fair standard 6-sided dice are thrown, the probability that the sum of the

numbers on the top faces is 10 can be written as $\frac{1}{6^n}$, where n is a positive integer. What is n ?

- (A) 42 (B) 49 (C) 56 (D) 63 (E) 84

11

answer is 84

0 0 0 0 0 0 0 0 0 0 ← 10
 9 gaps
 6 sticks
 $\frac{3 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = 12 \cdot 7 = 84$ ✓

Pat wants to buy four donuts from an ample supply of three types of donuts: glazed, chocolate, and powdered. How many different selections are possible?

- (A) 6 (B) 9 (C) 12 (D) 15 (E) 18

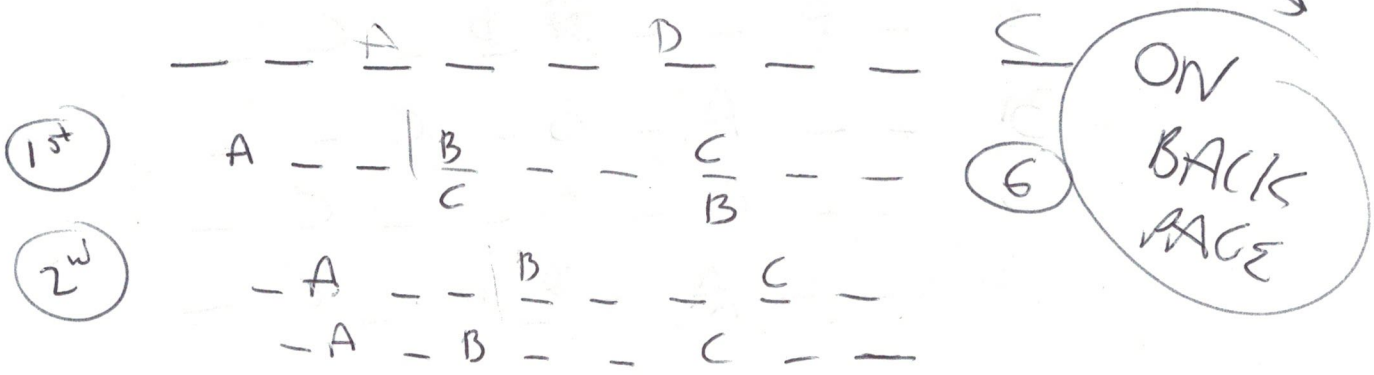
ans 15

$A + B + C = 4$, allow zero
 0 0 0 0
 7 stones
 6 gaps
 2 sticks
 $\frac{6 \cdot 5}{2} = 15$ ✓

Nine chairs in a row are to be occupied by six students and Professors Alpha, Beta and Gamma. These three professors arrive before the six students and decide to choose their chairs so that each professor will be between two students. In how many ways can Professors Alpha, Beta and Gamma choose their chairs?

- (A) 12 (B) 36 (C) 60 (D) 84 (E) 630

60



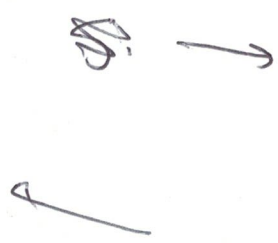
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Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

- (A) 22 (B) 25 (C) 27 (D) 28 (E) 729

28

6 stones
2 rub
5 sp



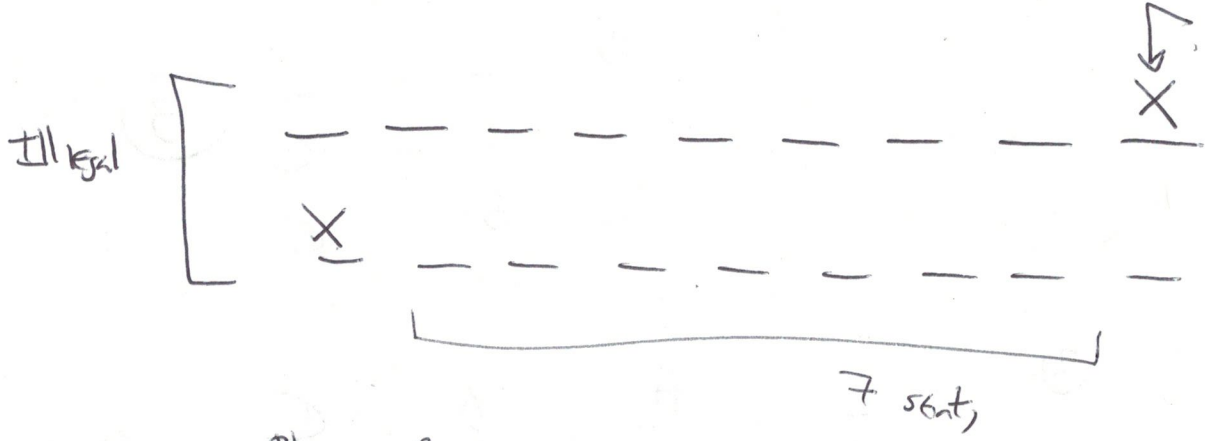
9 stones
8 gpp
2 rub

10. The telephone number problem. Non-decreasing 9-0, 8 digits

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9	9	8	8	7	7	7	7
9	9	7	8	8	7	4	2

$$\frac{87}{2} = 28 \checkmark$$



Place A, leaves
 $7 - 2 = 5$ seats



Place B, leaves $5 - 3 = 2$
or $5 - 2 = 3$

7

$$7 \times 1 \times 3 + 7 \times 4 \times 3$$

21 84