

Lesson 8: Repetition

Nikita, Konstantin Myagkov

Problem 1.

Can you cover the quadrilateral from Figure 1 by 1024 quadrilaterals (which are similar and 32 times smaller) from Figure 2?

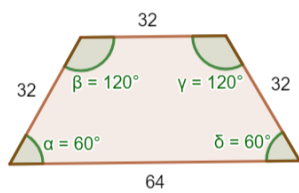


Figure 1

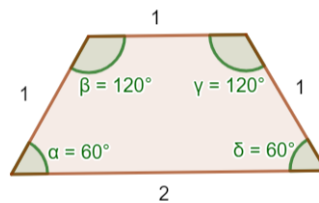


Figure 2

Problem 2.

Prove the identity for all $n \geq 1$:

$$\frac{1^2 + 1}{2} + \frac{2^2 + 2}{2} + \frac{3^2 + 3}{2} + \dots + \frac{n^2 + n}{2} = \frac{n(n + 1)(n + 2)}{6}$$

Problem 3.

In a rectangle $m \times n$ some cells are marked red. It is known that in each of m rows 10 cells are marked red and in each of n columns 8 cells are marked red.

Prove that m is divisible by 4 and n is divisible by 5.

Problem 4.

Triangle ABC is acute. The bisector of angle A and the perpendicular bisector of BC intersect at point D . Show that the $ABDC$ is cyclic.

Problem 5.

How many ways are there to split the 2×10 board into dominoes?

Problem 6.

a) Two squares of an 8×8 chess board have stones in them. With one move, it is possible to move one of the stones to an adjacent square. Is it possible to move the stones to a position symmetric to the initial position with respect to the central vertical line through the chessboard in exactly 2011 moves?

b) Same question, but with 5 stones and central symmetry.