Lesson 7: Inscribed angles

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Theorem 1.
Opposite angles of cyclic quadrilaterals sum to $180°$.

Theorem 2.
Conversely, any quadrilateral for which this is true can be inscribed in a circle.

Theorem 3.
If $ABCD$ is a convex quadrilateral with $\angle ACB = \angle ADC$, then it is cyclic.

Problem 0.
Let D be a point on a circumcircle of the equilateral triangle ABC. Prove that 2 of the three angles ADB, BDC and CDA are equal.

Problem 1.
The chords AB and CD intersect at a point M lying inside the circle. Prove that the triangles AMD and CMB are similar.

Problem 2.
Through one of the two intersection points of two circles, a diameter in each of the circles is drawn. Prove that the line connecting the endpoints of these diameters passes through the other intersection point.

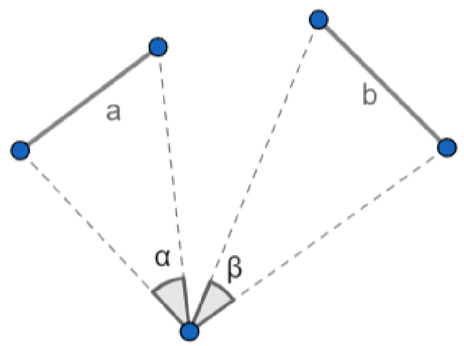
Problem 3.
Triangle ABC is acute. Prove that its circumscribed circle, bisector of angle A and the perpendicular bisector of BC all intersect at one point.

Problem 4.
The angle at the vertex A of a triangle ABC is $60°$. Prove that the center of the circle passing through the midpoints of the sides of triangle ABC, lies on the bisector of the angle A.

Problem 5.
Four points of the circle follow in order: A, B, C, D. The extensions of the AB chord beyond the point B and the CD chord beyond the point C intersect at the point E. The angle AED is $60°$. The angle ABD is three times greater than the angle BAC. Prove that AD is a diameter of the circle.

Problem 6.

Two segments a and b are given in the plane. Using a compass and a straightedge, draw a point from which one sees segment a under a given angle α, and segment b at a given angle β.



Problem 7.

Prove that given a triangle ABC and a point P on its circumcircle, the three closest points to P on lines AB, AC, and BC are collinear.