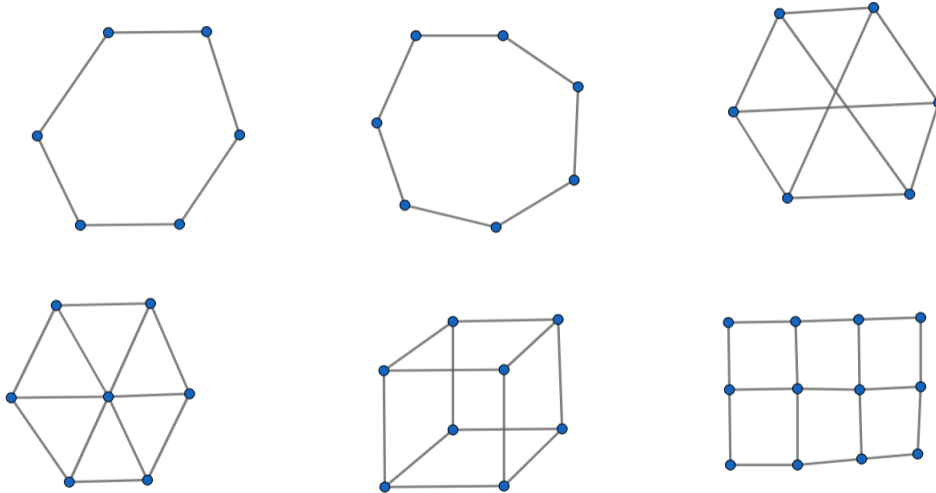


# Lesson 6: Bipartite Graphs and Geometry

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## Problem 0.

Are the following six graphs bipartite?



## Problem 1.

Show that a graph is bipartite if and only if it does not have odd cycles.

## Problem 2.

Show that a graph where every vertex has degree 2 is a collection of disjoint simple cycles.

## Problem 3.

On a test every student solved exactly 2 problems, and every problem was solved by exactly 2 students.

- Show that the number of students in the class and the number of problems on the test are the same.
- The teacher wants to make every student present one problem they solved on the board. Show that it is possible to choose the problem each student presents so that every problem on the test gets presented exactly once.

## Problem 4.

An angle bisector of the angle  $A$  of an acute triangle  $ABC$  intersects its circumcircle at the point  $D$ . Show that  $BD = CD$ .

## Problem 5.

- Let  $ABC$  be an acute triangle. Let  $AK$  and  $BL$  be altitudes, and call their intersection  $H$ . Show that  $\angle BLK = \angle HCK$
- Show that altitudes of an acute triangle intersect at one point.