

Lesson 5: Bipartite graphs and geometry

Konstantin Miagkov

Problem 1.

Is it possible to walk around the 7×7 chess board with a knight visiting every square exactly once and finishing back at the starting square?

Problem 2.

a) A bipartite graph has b white and r black vertices. What is the maximum possible number of edges in this graph?

b) What is the maximum possible number of edges in a bipartite graph with $2n$ vertices?

c) What about $2n + 1$ vertices?

Problem 3.

Show that if a graph has no cycles, then it is bipartite.

Problem 4.

In a quadrilateral $ABCD$ angles ABC and ADC are right. Also, $\angle ABD = 40^\circ$. Find $\angle CAD$.

Problem 5.

Given two circles with external tangency, prove that the common tangent passing through the tangency point, bisects the segments of external common tangents bounded by the tangency points.

Problem 6.

To two circles tangent externally at a point A , a common external tangent BC is drawn (where B and C are the tangency points). Prove that the angle BAC is right.

Hint: Draw through A a common tangent, define D and examine the triangles ABD and ADC .

Problem 7.

Let $ABCD$ be a cyclic quadrilateral with $AD = CD$. Let T be the intersection of lines AD and BC . Given that $AB = CT$, show that $\angle DBT = \angle DTB$.