Lesson 5: Bipartite graphs and geometry

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Problem 1.
Is it possible to walk around the $7 \times 7$ chess board with a knight visiting every square exactly once and finishing back at the starting square?

Problem 2.
a) A bipartite graph has $b$ white and $r$ black vertices. What is the maximum possible number of edges in this graph?
b) What is the maximum possible number of edges in a bipartite graph with $2n$ vertices?
c) What about $2n + 1$ vertices?

Problem 3.
Show that if a graph has no cycles, then it is bipartite.

Problem 4.
In a quadrilateral $ABCD$ angles $ABC$ and $ADC$ are right. Also, $\angle ABD = 40^\circ$. Find $\angle CAD$.

Problem 5.
Given two circles with external tangency, prove that the common tangent passing through the tangency point, bisects the segments of external common tangents bounded by the tangency points.

Problem 6.
To two circles tangent externally at a point $A$, a common external tangent $BC$ is drawn (where $B$ and $C$ are the tangency points). Prove that the angle $BAC$ is right.
   Hint: Draw through $A$ a common tangent, define $D$ and examine the triangles $ABD$ and $ADC$.

Problem 7.
Let $ABCD$ be a cyclic quadrilateral with $AD = CD$. Let $T$ be the intersection of lines $AD$ and $BC$. Given that $AB = CT$, show that $\angle DBT = \angle DTB$. 

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