

Futoshiki

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1. Futoshiki is a game played on a square grid of size $N \times N$. The aim of Futoshiki is to place the numbers 1 to N into each row and column of the puzzle so that no number is repeated in a row or column and so that all of the inequality signs ($<$ and $>$) are obeyed. Solve these two puzzles.

1	3	2
3	2	1
2	1	3

1	4	>	3	2	
3	2		1	4	
2	1		4	3	
4	>	3	>	2	1

2. Complete the following row in a Futoshiki Square using the numbers 1, 2, and 3.

3	>	2	>	1
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3. Complete the following row in a Futoshiki Square using the numbers 1, 2, 3, and 4. The first two squares must have either 1 or 2 in them.

1 /	2	1 /	2	3	>	4
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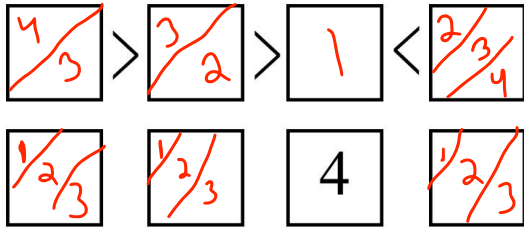
4. More than two numbers may be possible in a single square. For example to solve the following row

		4	
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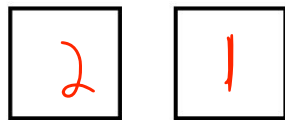
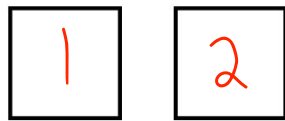
we write the numbers as shown below.

1/2/3	1/2/3	4	1/2/3
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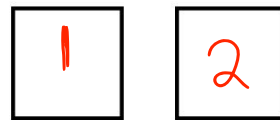
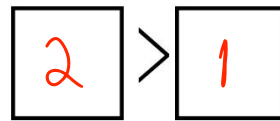
5. The first two rows of a Futoshiki square are shown below. Complete the rows. If more than one number is possible in a square, write all possible numbers in small script as shown above (these are notes).



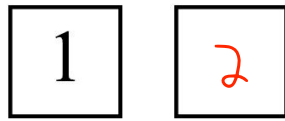
6. A good Futoshiki board only has one solution. How many different ways can you solve these 2×2 boards? Show one solution on each board.



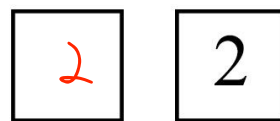
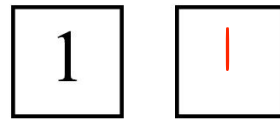
2 solutions



1 solutions

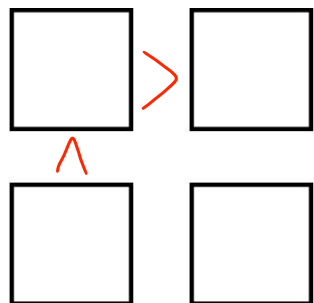


1 solutions



0 solutions

7. Create a 2×2 Futoshiki board with no solutions. Do not use any numbers to create the board. Use only inequalities.



8. How many different ways can you solve the first row of a 3×3 Futoshiki square with no inequalities?

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$$\underline{3} \quad \underline{2} \quad \underline{1} = 6 \text{ solutions}$$

9. How many different ways can you solve the second row of a 3×3 Futoshiki square with no inequalities if the first row is already filled out?

1	2	3

$$\underline{2} \quad \underline{1} \quad \underline{1} = 2 \text{ solutions}$$

10. How many different ways can you solve the third row of a 3×3 Futoshiki square with no inequalities if the first and second row are already filled out?

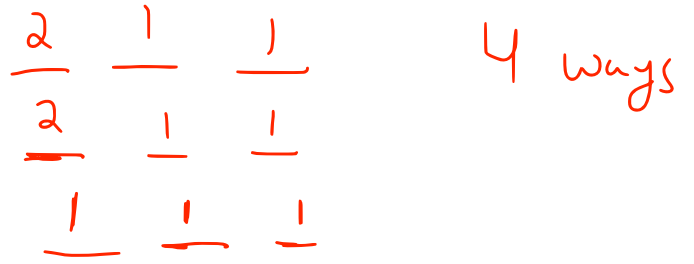
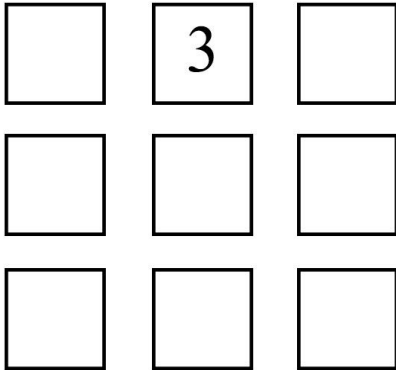
1	2	3
2	3	1

$$\underline{1} \quad \underline{1} \quad \underline{1} = 1 \text{ solution}$$

11. How many ways can you solve a 3×3 Futoshiki square with no inequalities?

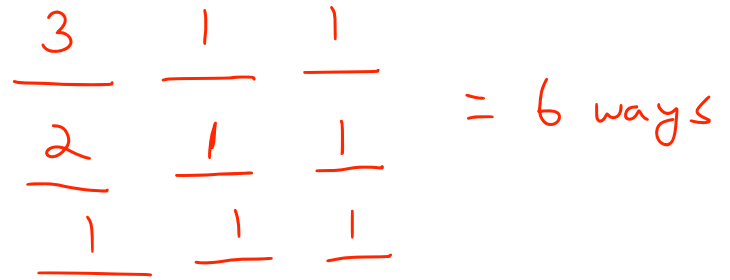
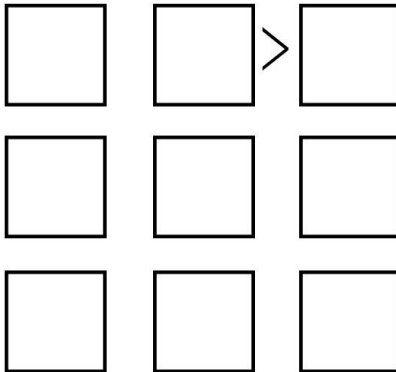
$$\underline{6} \quad \underline{2} \quad \underline{1} = 12 \text{ ways}$$

12. How many ways can you solve a 3×3 Futoshiki square with one number given? Does it matter where the number is?



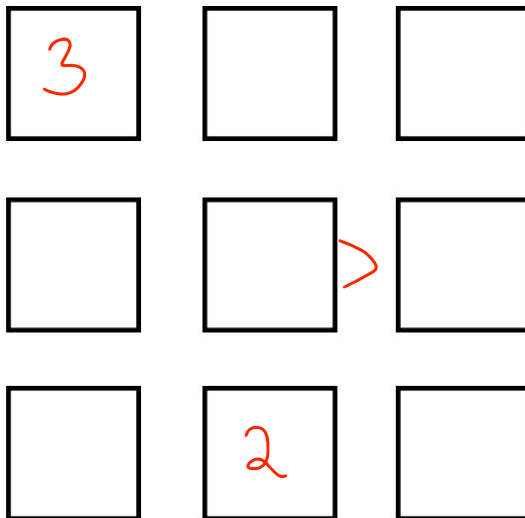
It does not matter where the number is.

13. How many ways can you solve a 3×3 Futoshiki square with one inequality? Does it matter where the inequality is?

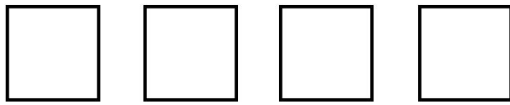


It does not matter where the inequality is.

14. Design a 3×3 Futoshiki puzzle with exactly one solution. Try to use as few initial numbers and inequalities as possible. Put your final puzzle on the separate template provided. Exchange boards with your partner and solve each other's puzzle.

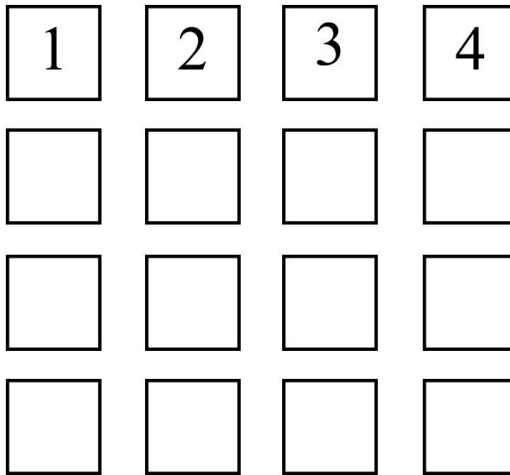


15. How many ways can you solve the first row of a blank 4×4 Futoshiki board?



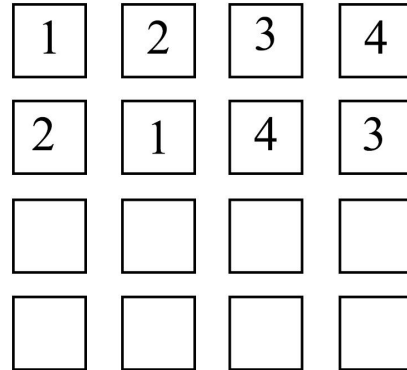
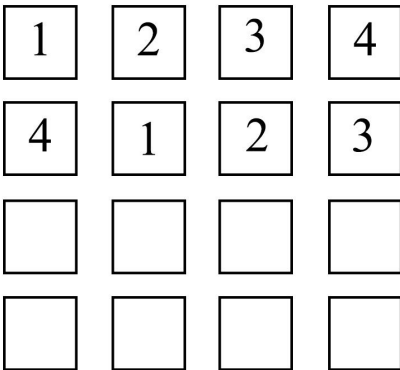
$$\underline{4} \quad \underline{3} \quad \underline{2} \quad \underline{1} = 24 \text{ ways}$$

16. How many ways can you solve the second row of a blank 4×4 Futoshiki board with the first row already completed?



$$\underline{3} \quad \underline{2} \quad \underline{1} \quad \underline{1} = 6 \text{ ways}$$

17. How many ways can you solve the third row of a blank 4×4 Futoshiki board with the first two rows already completed? Consider these two cases to help you.



$$\underline{2} \quad \underline{1} \quad \underline{1} \quad \underline{1} = 2 \text{ ways}$$

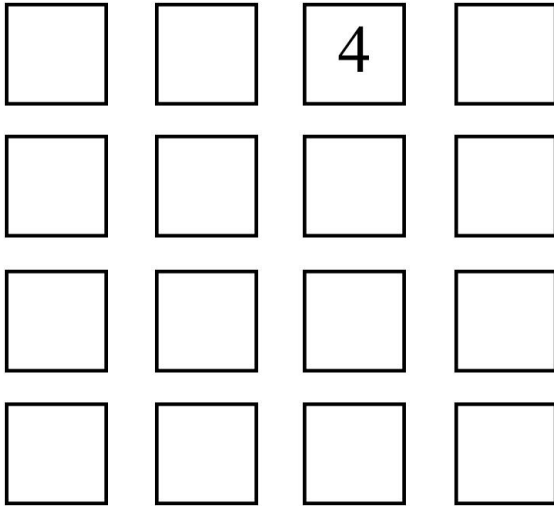
$$\left. \begin{array}{l} 4 \text{ ways} \\ \underline{2} \quad \underline{1} \quad \underline{2} \quad \underline{1} \end{array} \right\}$$

18. How many ways can you solve a blank 4×4 Futoshiki board?

$$576 \text{ ways}$$

$$\underline{24} \quad \underline{6} \quad \underline{4} \quad \underline{1} =$$

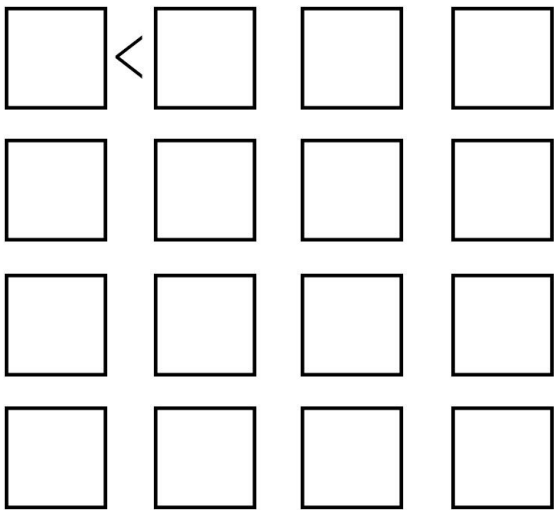
19. How many ways can you solve a 4×4 Futoshiki square with one number given? Does it matter where the number is?



$$\begin{array}{cccc}
 \underline{3} & \underline{2} & \underline{1} & \underline{1} = 6 \\
 \underline{3} & \underline{2} & \underline{1} & \underline{1} = 6 \\
 \underline{2} & \underline{1} & \underline{1} & \underline{1} = 2 \\
 \underline{1} & \underline{1} & \underline{1} & \underline{1} = 1 \\
 \hline
 = 6 \cdot 6 \cdot 2 \cdot 1 = 72
 \end{array}$$

It does not matter where the number is.

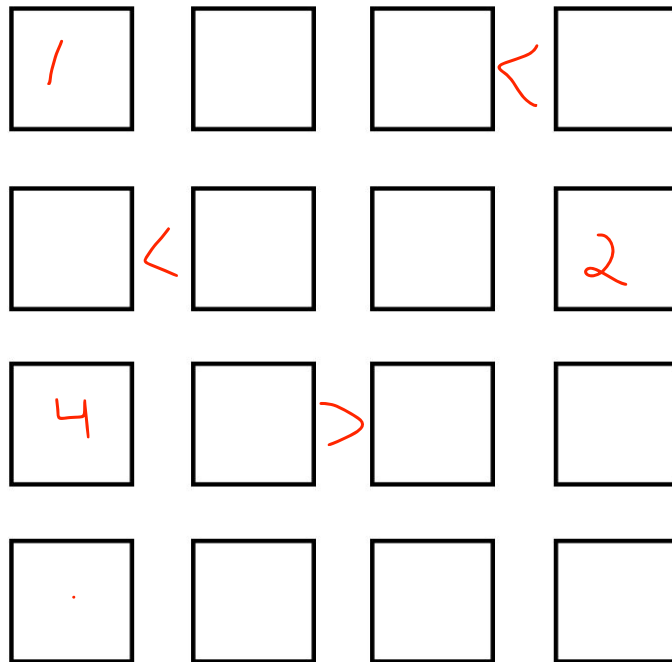
20. How many ways can you solve a 4×4 Futoshiki square with one inequality? Does it matter where the inequality is?



$$\begin{array}{cccc}
 \underline{3} & \underline{2} & \underline{2} & \underline{1} = 12 \\
 \underline{3} & \underline{2} & \underline{1} & \underline{1} = 6 \\
 \underline{2} & \underline{1} & \underline{1} & \underline{1} = 2 \\
 \underline{1} & \underline{1} & \underline{1} & \underline{1} = 1 \\
 \hline
 = 12 \cdot 6 \cdot 2 \cdot 1 = 144
 \end{array}$$

It does not matter where the inequality is.

21. Design a 4×4 Futoshiki puzzle with exactly one solution. Try to use as few initial numbers and inequalities as possible. Put your final puzzle on the separate template provided and exchange your puzzle with a partner.



Challenge Puzzles

