

TRANSFORMATIONS VIA PERMUTATIONS

JUNIOR CIRCLE 05/08/2011

We will look at the following transformations of an equilateral triangle:

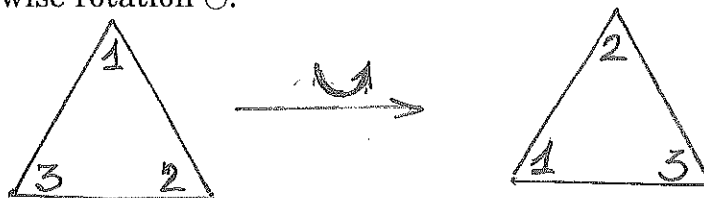
- Rotations;
- Reflections (flips) in a line;

The two types of rotation are:

- Clockwise rotation \circlearrowright :



- Counterclockwise rotation \circlearrowleft :



(1) When the triangle is rotated, the vertices end up in the new places. This way, we get a permutation of vertices:

- The first row is starting positions;
- The second row is ending positions;

Write down the permutations corresponding to the clockwise and the counterclockwise rotations:

(a) Clockwise rotation \circlearrowright :

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

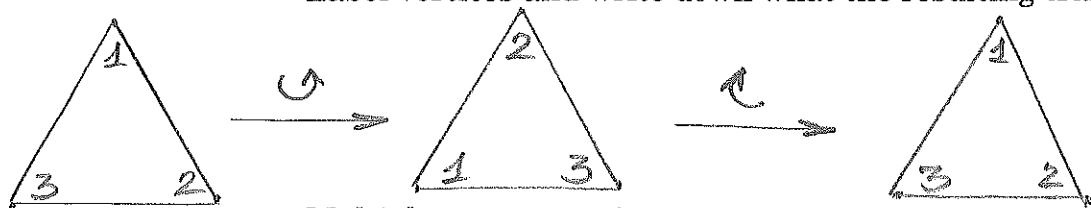
(b) Counterclockwise rotation \circlearrowleft :

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

(3) Let's find the result of performing two transformations in a row:

(a) Find what transformation $\circlearrowleft \circ \circlearrowleft$ equals to in two different ways:

- Label vertices and write down what the resulting transformation is:



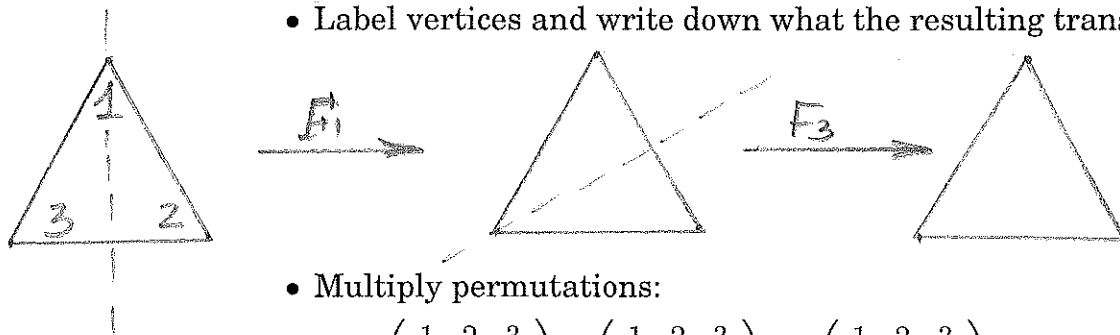
- Multiply permutations:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

- Are the results you get when using the picture and when multiplying permutations the same?

(b) Find what transformation $F_1 \circ F_3$ equals to in two different ways:

- Label vertices and write down what the resulting transformation is:



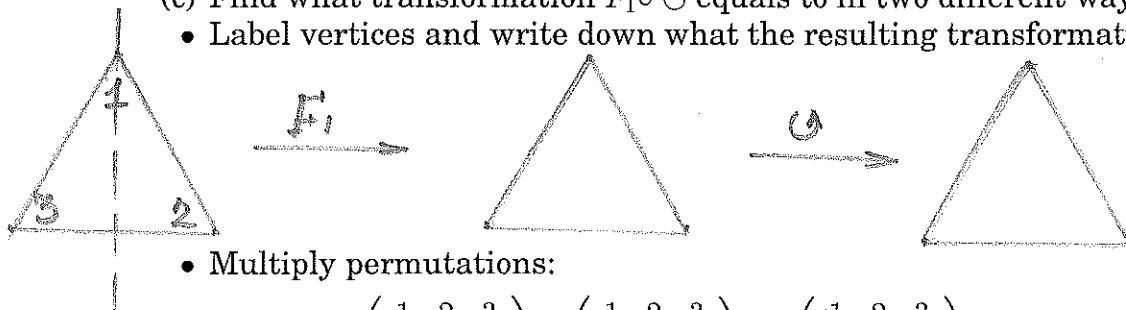
- Multiply permutations:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

- Are the results you get when using the picture and when multiplying permutations the same?

(c) Find what transformation $F_1 \circ \circlearrowleft$ equals to in two different ways:

- Label vertices and write down what the resulting transformation is:



- Multiply permutations:

$$\begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix};$$

- Are the results you get when using the picture and when multiplying permutations the same?