

Understanding Trade Part II

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Math Circle

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Last week, we introduced the basic idea of trade. We thought about two islanders who each had their own rates of apple-picking and bread-baking, and we explored some numeric examples designed to show that trade can be a win-win interaction. We looked at some intuitive cases, and we also looked at one surprising case: trade can be win-win even if one islander was faster than his neighbor at both apple-picking and bread-baking.

This week, I want to review that material but using a few new tools, including a cool graphical representation that I am borrowing from Khan Academy's AP Economics video series.

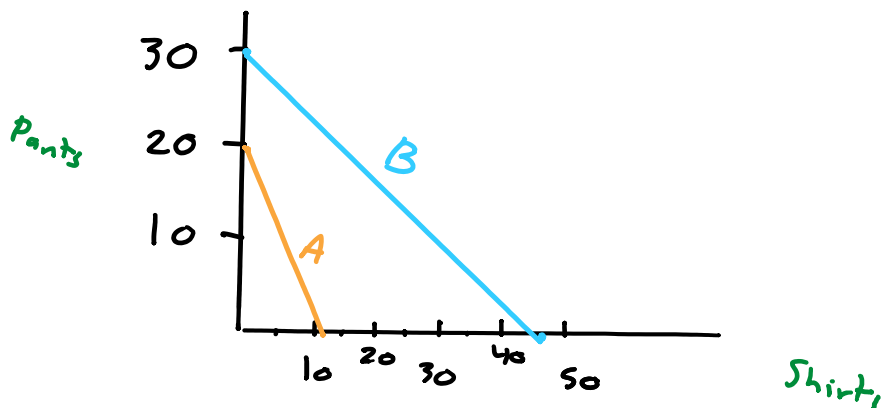
1. Country A and Country B can each produce two items: pants and shirts.
2. The workers in Country A can produce 2 pairs of pants an hour, or they can produce 1 shirt per hour. The workers in Country B are a little faster. They can produce 3 pairs of pants per hour, or they can produce 4.5 shirts per hour. We can think of these as their rates of production, and we can capture them in a simple table:

	A	B
Pants	2	3
Shirts	1	4.5

3. In a 10-hour day, Country A could choose to make only pants, only shirts, or some combination. For example, Country A could choose to spend all 10 hours making pants, which would yield 20 pairs of pants. Or Country A could instead choose to spend all 10 hours making shirts, which would yield 10 shirts. Or Country A could choose some intermediate option, like spending 5 hours making pants (10 pairs) and 5 hours making shirts (5 shirts)
4. Country B similarly has choices that range from 30 to 0 pairs of pants, and correspondingly from 0 to 45 shirts. For example, Country B investing all 10 hours on pant production would result in 30 pairs of pants but zero shirts.

5. We can capture these opportunity sets in a graph.

5a. The x-axis counts shirts. The y-axis counts pants. And then I separately marked A's opportunity set in orange, and B's opportunity set in blue.



5b. Note that A cannot achieve any outcomes to the right of its line. That is, no matter what, A cannot produce (say) 20 shirts plus 5 pants in a day. The line is A's set of maximum choices. A can do less, for example by taking a few hours for vacation. But A cannot exceed that line. Similarly, B cannot exceed its line.

6. We can now examine some of the most important relationships here by creating an output table showing the maximum number of pants and shirts produced in each country.

	A	B
Max Pants	20	30
Max Shirts	10	45

7. Using the above table, think about the opportunity costs associated with making pants. That is, what does Country A give up in order to make a pair of pants?

7a. Start with time. How much time must Country A invest to make a pair of pants?

30 minutes

7b. Now think about shirts. By making a pair of pants, Country A gives up the opportunity to make how many shirts?

$\frac{1}{2}$ of a shirt

7c. Now focus on Country B. How much time must Country A invest to make a pair of pants?

20 minutes

7d. Okay, now measure in terms of shirts. How many shirts does Country A give up in order to make a pair of pants?

1.5 shirts

$$\begin{aligned} 4.5 \text{ shirts} &= 1 \text{ hour} \\ 1.5 &= \frac{1}{3} \text{ hour} \end{aligned}$$

7e. Knowing these costs, which country should make pants? That is, making pants is "cheaper" for one of these countries than it is for the other. Which country makes pants more cheaply?

Country A $\frac{1}{2}$ shirt vs. $\frac{3}{2}$ shirt

8. Now let's do the same analysis, but focused on shirts. Any predictions about what we will see?

A makes 1 shirt at cost of 2 pants

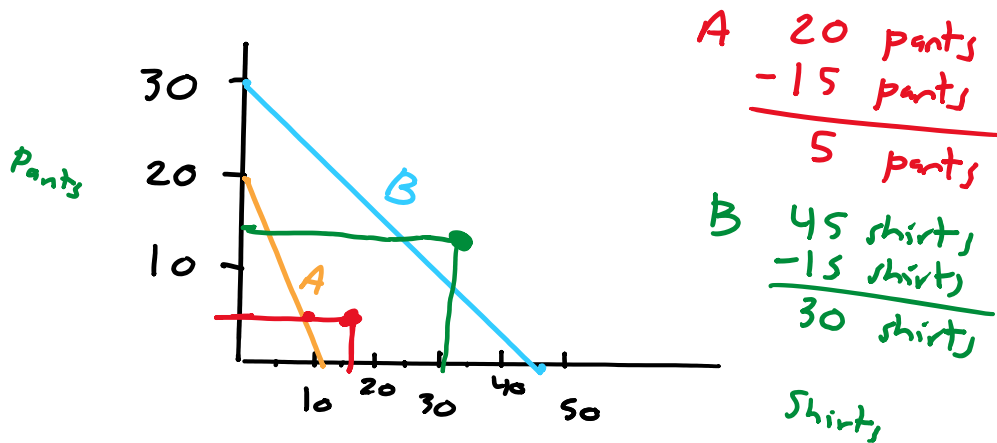
B makes 1 shirt at cost of $\frac{2}{3}$ pants

These are the inverses of $\frac{1}{2}$ and $\frac{3}{2}$!!

9. If Country A and Country B are going to engage in trade, what is the lowest price that Country A would be willing to sell pants? What is the highest price that Country B would pay for a pair of pants? Is there room for a deal?

$\frac{1}{2}$ shirt \longrightarrow $\frac{3}{2}$ shirt \longleftarrow Highest
Lowest \longleftarrow
Maybe pick price of 1 shirt = 1 pant

10. Suppose that Country A gives 15 pairs of pants to Country B, and Country B in exchange gives Country A 15 shirts. Add these trades to our graph. What do you see?



Wow! Both countries reach points they could not have otherwise reached!

11. Now we are ready to answer some hard questions about trade. First, when will there be no gains from trade?

11a. Again imagine that Country A and Country B can each produce pants and shirts.

11b. The workers in Country A can produce 2 pairs of pants an hour, or they can produce 1 shirt per hour. The workers in Country B are a little faster. They can produce 4 pairs of pants per hour, or they can produce 2 shirts per hour.

11c. Now think about the opportunity costs associated with making pants. That is, what does Country A give up in order to make a pair of pants? Country B?

A gives up $\frac{1}{2}$ shirt $\leftarrow \begin{array}{l} 2 \\ 1 \end{array} \frac{1}{2}$

B gives up $\frac{1}{2}$ shirts $\leftarrow \begin{array}{l} 4 \\ 1 \end{array} \frac{2}{2}$

11d. Do you see why, in this example, there are no gains from trade? Both countries must invest half a shirt in order to produce one pair of pants. So there is no advantage to trading; their costs are the same.

12. Are there other examples where there would be no gains from trade?

12a. Go back to our first example where the workers in Country A can produce 2 pairs of pants an hour, or they can produce 1 shirt per hour, but the workers in Country B can produce 3 pairs of pants per hour, or they can produce 4.5 shirts per hour.

12b. Last time, we found room for trade because the countries have different opportunity costs. But what happens if Country A does not value pants? Is there still room for trade?

Yes! A makes pants, trades 1-for-1 with B, and is better off.

12c. Try the opposite assumption. What happens if Country B does not value pants? Is there still room for trade?

No. "I make pants cheap!" says A.
"Who cares?" responds B. "I do not like pants."

13. DISCUSS. How helpful is this simple model? Math is supposed to help us understand the real world. Does this model do that? What important things matter in the real world but might not be captured in the above examples?

Some ideas: there are costs to shipping; countries might want to produce pants for themselves, worried about disruptions in supply in the event of a war or natural disaster; countries might have trouble agreeing on a price, even if there is a price that would create a win-win trade.