

## Understanding Trade

Doug Lichtman

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Last week, we talked about the classic formula "distance equals rate times time" and we applied that formula to a variety of examples. This week, we took that same formula and used it to think about an important real-world issue: trade.

### **ALEX THE ISLANDER**

Suppose that Alex lives all alone on an island, and that he has two basic skills: he can pick apples, and he can bake bread. Indeed, suppose that Alex can pick 10 apples per hour and Alex can bake one loaf of bread per hour. Knowing just those facts, and assuming that Alex works eight hours per day, we can make a chart that shows the universe of choices that Alex faces each lonely day:

<u>Hours spent picking apples</u>	<u>Hours spent baking bread</u>	<u>Apples</u>	<u>Loaves</u>
8	0	80	0
7	1	70	1
6	2	60	2
5	3	50	3
4	4	40	4
3	5	30	5
2	6	20	6
1	7	10	7
0	8	0	8

Just to make sure we are reading the chart the same way, consider the row where Alex spends 4 hours picking apples. Because he can work only eight hours, the fact that Alex spent 4 hours picking apples means that he has 4 hours to bake bread. And, because he picks 10 apples per hour, his picking nets him 40 apples. Likewise, because he bakes one loaf of bread per hour, his baking nets him 4 loaves.

## **BEN THE ISLANDER**

Now consider Ben. Like Alex, Ben lives on an island all by himself, and Ben can only pursue two activities: he can pick apples, or he can bake bread. Unlike Alex, however, Ben is a very bad picker of apples and a very good baker. So Ben can only pick 1 apple an hour, but he can bake a lightning-fast 10 loaves of bread per hour. Ben's opportunity set looks like this:

<u>Hours spent picking apples</u>	<u>Hours spent baking bread</u>	<u>Apples</u>	<u>Loaves</u>
8	0	8	0
7	1	7	10
6	2	6	20
5	3	5	30
4	4	4	40
3	5	3	50
2	6	2	60
1	7	1	70
0	8	0	80

## **THE CHOICES LONELY ISLANDERS MAKE**

In a world without trade, Alex will have to decide for himself how to spend his day. If he hates apples but loves bread, the choice is easy: he should spend the entire day baking bread. But if he values both apples and bread, he will have to make a decision about how many hours to spend pursuing each activity. There is no right or wrong answer; this is just Alex's personal preference about what he wants to eat.

Similarly, in a world without trade, Ben will have to decide for himself how to spend his day. Like Alex, Ben can focus on bread-baking if he likes only bread, or he can focus on apple-picking if he likes only apples. But if he likes both, he will have to look at the chart and make a decision about how to spend his time.

Suppose that Alex chooses the row where he spends 5 hours picking apples, and suppose that Ben chooses the row where he spends 6 hours picking apples. That means that Alex has 50 apples and 3 loaves of bread, while Ben has 6 apples and 20 loaves of bread.

The question for us: if Alex and Ben could knock down a tree, connect their islands by that make-shift bridge, and trade, would they possibly be better off?

## LOOKING AT POSSIBLE TRADES

1. Imagine that Alex decides to focus fully on picking apples, given that he is so fast. He can eat some and trade the rest. How many apples would Alex pick?
2. Now imagine that Ben decides to focus fully on baking bread, given that he is so fast. He too can eat some and trade the rest. How many loaves would he bake?
3. Taken together, how many apples would Alex and Ben as a group have? And how many loaves of bread?
4. Can you propose a trade that would make both Alex and Ben better off? That is, before they engaged in trade, Alex would have worked all day and had 50 apples and 3 loaves of bread, while Ben would have worked all day and had 6 apples and 20 loaves of bread. Now, if Alex spends all his time picking apples and Ben spends all of his time baking bread, would they at the end of the day be able to make a trade that would leave both Alex and Ben even happier? Could Alex end up with at least 50 apples and at least 3 loaves of bread, while Ben ends up with at least 6 apples and at least 20 loaves of bread? Can they each end up with more?
5. Suppose that Alex does not want to spend his entire day picking apples. Is there still room for a win-win trade? That is, does Alex have to specialize in apple-picking for the math to work, or can he still do some bread-baking, too, just to stay in practice?

## VARIATIONS ON RATES

In the first example, Alex and Ben were polar opposites. Alex picked 10 apples per hour, and Ben baked 10 loaves per hour. Alex baked 1 loaf of bread per hour, and Ben picked one apple per hour. Does trade still produce win-win outcomes if the islanders are not polar opposites?

6. For example, what would happen if Alex picked 5 apples per hour and baked 2 loaves of bread per hour, whereas Ben picked 2 apples per hour and baked 2 loaves of bread per hour? Would there still be benefits to trade? Make the charts and see if trade still produces win-win outcomes for Alex and Ben.

7. In both of the prior examples, Alex was better than Ben at one activity but not both. But think now about a situation where Alex is better at everything. For example, suppose that Alex can pick 6 apples per hour or bake 2 loaves of bread per hour, whereas Ben can only pick 1 apple per hour or bake one loaf of bread per hour. Note that Alex is faster at apple-picking, and Alex is faster at bread-baking. Can Alex still benefit himself by trading with the slower Ben? Again, make the charts and explore some trades. Any surprises?

## HOMEWORK

8. Are there any examples where trade does not benefit Alex and Ben? That is, can you propose a set of rates (we need four: Alex's apple rate, Alex's bread rate, Ben's apple rate, and Ben's bread rate) that would cause Alex to not want to trade with Ben? Can you explain why the rates you chose fail to encourage trade? Assume that Alex and Ben both like apples, and Alex and Ben both like bread.