

# Distance formula

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## Introduction to the Distance Formula

1. Jereth rides a bike with the speed of 8 miles per hour. How far will he get in 3 hours?

$$8 \text{ miles/hour} * 3 \text{ hours} = 24 \text{ miles}$$

2. Nicole is training for a marathon. It takes her 2 hrs to run 6 miles. What is the speed at which she runs?

$$\begin{aligned} 6 \text{ miles}/2 \text{ hours} &= 3 \text{ miles/hour} \\ 3 \text{ miles/hour} &= 3 \text{ miles}/60 \text{ mins} = 1 \text{ mile}/ 20 \text{ mins} \end{aligned}$$

3. Kate is running at a speed of 6 km per hour. How long will it take her to run 12 km?

$$12 \text{ km} * \text{hour}/6\text{km} = 2 \text{ hours}$$

4. Leo is at home and rides his bike to the store. He rides to the store at 8 miles per hour for 2 hours. On the way back home, he is tired and decreases his speed by 2 miles per hour. How long did it take him to go to the store and back?

distance one way = 8 miles/hour \* 2 hours = 16 miles

on the way back: rate of  $8-2=6$  miles/hour  
 $16 \text{ miles} * 1 \text{ hour}/6 \text{ miles} = 8/3$  hours, or 2 hours and 40 mins

5. A motor boat travels 104 km in 4 hrs. How long will it take to travel 174 km if the boat increases its speed by 3 km/hr?

rate per hour:  $104 \text{ km}/4\text{hrs} = 26 \text{ km/hr}$   
new rate:  $26+3 = 29 \text{ km/hr}$

$174 \text{ km} * 1\text{hr}/29\text{km} = 6 \text{ hours}$

6. A car goes with the speed of  $v$  mph. Let  $S$  be the distance that the car covers in  $t$  hours.

(a) Write down a formula expressing  $S$  in terms of  $v$  and  $t$ :

$$S = v*t$$

(b) Rewrite the formula expressing  $v$  in terms  $S$  and  $t$ :

$$v = S/t$$

(c) Now write the formula expressing  $t$  in terms of  $v$  and  $S$ .

$$t = S/v$$

7. Lucy drives her car at a speed of  $v$  km per hour. How long will it take her to drive 100 km?

$$100 \text{ km} * 1 \text{ hr/vkm} = 100/v \text{ hours}$$

8. Kaley went on a plane to travel to New York. Her flight took 5 hours. The distance between Los Angeles and New York is  $S$  miles. At what speed was the plane flying?

$$s \text{ miles} / 5 \text{ hours} = s/5 \text{ miles/hr}$$

9. Emmanuelle goes for a jog around the neighborhood. For the first hour, she runs at a speed of  $v$  miles per hour. For the next two hours, she runs at  $u$  miles per hour. How far does she run?

$$v \text{ miles/hr} * 1 \text{ hr} + u \text{ miles/hr} * 2\text{hrs} = v + 2u \text{ miles}$$

## Two Moving Objects

Please make a picture (diagram) in each of the problems below.

1. Ariella and Anna walk towards each other along the same street starting at their houses. Ariella's speed is 50 meters per minute, and Anna's speed is 40 meters per minute. The distance between their houses is 360 meters. They began walking at the same time.

(a) How long do they have to walk before they meet?

$$50 \text{ m/min} + 40 \text{ m/min} = 90 \text{ m/min}$$

$$360 \text{ m} \cdot 1 \text{ min}/90\text{m} = 4 \text{ mins}$$

(b) How far will Anna be from her house when they meet?

$$40 \text{ m/min} \cdot 4 \text{ mins} = 160 \text{ m}$$

2. Asher and Aaron start walking from the same point in opposite directions. Asher's speed is 30 meters per minute. Aaron's speed is 40 meters per minute.

(a) How far apart will they be in 8 minutes?

$$30 \text{ m/min} + 40 \text{ m/min} = 70 \text{ m/min}$$

$$70 \text{ m/min} \cdot 8 \text{ mins} = 560 \text{ m}$$

(b) How long will it take them to reach the distance of 630 meters?

$$630 \text{ m} * 1 \text{ min}/70 \text{ m} = 9 \text{ mins}$$

3. Prakash and Ethan are riding bikes on the same bike trail. At first, Prakash is ahead of Ethan by 50 meters. Prakash is riding his bike at a speed of 3 meters per second, and Ethan is riding his bike at 5 meters per second.

(a) How long will it take Ethan to catch up to Prakash?

$$\text{Ethan rides } (5-3) = 2 \text{ m/sec faster}$$

$$50 \text{ m} * 1 \text{ sec}/2 \text{ m} = 25 \text{ seconds}$$

(b) How far does Ethan travel in order to catch up to Prakash? How far does Prakash travel during this time period?

$$\text{Ethan: } 25 \text{ secs} * 5 \text{ m/sec} = 125 \text{ meters}$$

$$\text{Prakash: } 25 \text{ secs} * 3 \text{ m/sec} = 75 \text{ meters}$$

4. August and Asher are also riding their bikes on the same bike trail. August is riding his bike at 2 meters per second, and Asher is riding his bike at 4 meters per second. Asher is originally ahead of August by a distance of  $d$  meters. After 10 seconds, Asher is ahead of August by 40 meters.

(a) How far was Asher originally ahead of August? (What is  $d$ ?)

Asher travels  $(40-d)$  meters more than August in 10 secs  
Asher travels  $(4-2) = 2$  m/sec faster than August

So,  $2 \text{ m/sec} * 10 \text{ secs} = 40-d$   
 $20 \text{ meters} = 40-d$   
 $d - 20 \text{ meters}$

(b) After  $t$  seconds, Asher is ahead of August by 100 meters. What is  $t$ ?

Asher travels an extra  $(100-20) = 80$  meters than August

So,  $80 \text{ m} * 1 \text{ sec}/2\text{m} = 40 \text{ seconds}$ .

5. Dimitri and Michael are running on the same street. Michael is originally ahead of Dimitri by a distance of  $d$  meters. Michael is running at a speed of  $w$  meters per second, and Dimitri is running at a speed of  $u$  meters per second. Assume that Dimitri's speed is less than Michael's speed.

(a) Since Michael is ahead of Dimitri in the beginning and runs with a faster speed, the distance between them will be increase. *Relative speed*  $v$  is the rate at which the distance increases. By how many meters does the distance between Michael and Dimitri increase? Set this expression equal to  $v$ .

$$v = (w-u) \text{ m/sec}$$

(b) Using the speed of separation,  $v$ , what is the distance between Dimitri and Michael  $t$  seconds after they start running?

$$\begin{aligned} \text{increased distance} &= v \text{ m/sec} * t \text{ secs} \\ &= v * t \text{ meters} \end{aligned}$$

(c) How far apart are Dimitri and Michael in  $t$  seconds? Set this expression equal to  $S$ , which represents the total distance between Michael and Dimitri.

$$\begin{aligned} \text{distance between} &= \text{starting distance} + \text{increased distance} \\ S &= d \text{ m.} + v \text{ m/sec} * t \text{ secs} \\ S &= (d + v * t) \text{ meters} \end{aligned}$$

(d) Assume that  $v$  is a negative value. Does this mean that Dimitri is approaching or separating from Michael? As time goes by, will  $S$  increase or decrease?

Dimitri is approaching Michael. As time goes by,  $S$  will first decrease until it reaches 0, and will start increasing again.

## Challenge problems

1. It takes Lev 20 minutes to walk from his house to the store. It takes Richard 40 minutes to walk from Lev's house to the store. One morning, Richard left from Lev's house to the store 10 minutes before Lev left. How long will it take Lev to catch up with Richard?

let  $d$  = distance from Lev's house to the store. in meters  
Then, Lev's rate is  $d/20$  meters/min and  
Richard's rate is  $d/40$  meters/min.

Let  $t$  = time it takes for Lev to catch up to Richard.  
 $d/20 \text{ m/min} * t \text{ mins} = d/40 \text{ m/min} * (t+10) \text{ mins}$   
 $d*t/20 \text{ m} = d*t/40 \text{ m} + d/4 \text{ m}$   
 $d*t/40 = d/4$   
 $t/40 = 1/4$   
 $t = 40/4$   
 $t = 10 \text{ mins}$

It will take Lev 10 mins to catch up to Richard.

2. It takes Joy 30 minutes to go from school to the park. It takes Lola 40 minutes to go from school to the park. One afternoon, Lola got out of the school 5 minutes before Joy. How long will it take Joy to catch up with Lola?

$d/30 \text{ m/min} * t \text{ mins} = d/40 \text{ m/min} * (t+5) \text{ mins}$   
 $d*t/30 \text{ m} = dt/40 \text{ m} + d/8 \text{ m}$   
 $d*t/120 \text{ m} = d/8 \text{ m}$   
 $t/120 = 8$   
 $t = 120/8 = 15$

It will take Joy 15 mins to catch up to Lola.



3. Lydia and Emmie are running a mile. Lydia runs with a constant speed. For the first half of the distance, Emmie runs at half Lydia's speed. For the second half of the distance, Emmie runs twice as fast as Lydia. Who will finish the race first?

let  $L$  = rate Lydia runs in miles/min

Emmie runs at a rate of  $L/2$  miles/min in the first half mile and then  $2L$  miles/min in the second half

Lydia's time:  $1 \text{ mile} * 1/L \text{ mins/mile} = 1/L \text{ mins}$

Emmie's time:  $1/2 \text{ mile} * 2/L \text{ mins/mile} + 1/2 \text{ mile} * 1/2L \text{ mins/mile} = 1/L \text{ mins} + 1/4L \text{ mins} = 5/4L \text{ mins}$

Lydia finishes first.

4. Lydia and Emmie decide to race each other again in the mile. Lydia runs with a constant speed. This time, for the first half of the distance, Emmie runs at twice Lydia's speed. For the second half of the distance, Emmie runs at half Lydia's speed. Who will finish the race first?

Lydia will still finish the race first. The total time it takes for them to run a mile is still the same proportion.