

Lesson 1: Induction I

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Problem 1.

Show that the number $111\dots 111$ consisting of 243 ones is divisible by 243. Hint: $243 = 3^5$. This problem can be generalized as follows: for any positive integer n , the number consisting of 3^n ones is divisible by 3^n .

Problem 2.

a) Show that

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

b) Show that

$$1 + 3 + 5 + \dots + (2n-1) = n^2$$

Problem 3.

Show that $n^3 + 2n$ is divisible by 3 for any positive integer n .

Problem 4.

a) Show that for any positive integer n we have $2^n > n$.

b) Find all positive integers n such that $2^n > n^2$.

Problem 5.

Suppose there are n lines drawn on a plane, in such a way that not two lines are parallel and no three lines intersect at the same point. Find a closed formula for the number of regions in which the lines split the plane.