Lesson 3: Induction in Algebra

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Problem 1.
Show that

$$1 + 3 + 5 + \ldots + (2n - 1) = n^2$$

Problem 2.
Show that

$$2 + 5 + 8 + \ldots + (3n - 1) = \frac{3n^2 + n}{2}$$

Problem 3.
Show that

$$1 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Problem 4.
Show that

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n \times (n+1) = \frac{n(n+1)(n+2)}{3}$$

Problem 5.
Show that

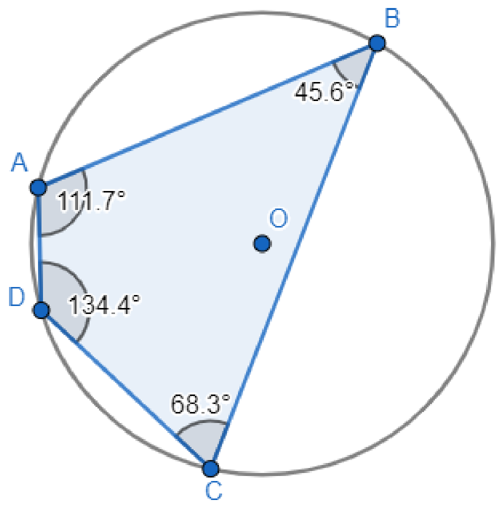
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{(n-1) \cdot n} = \frac{n-1}{n}$$

Problem 6 (*)**.**
Show that

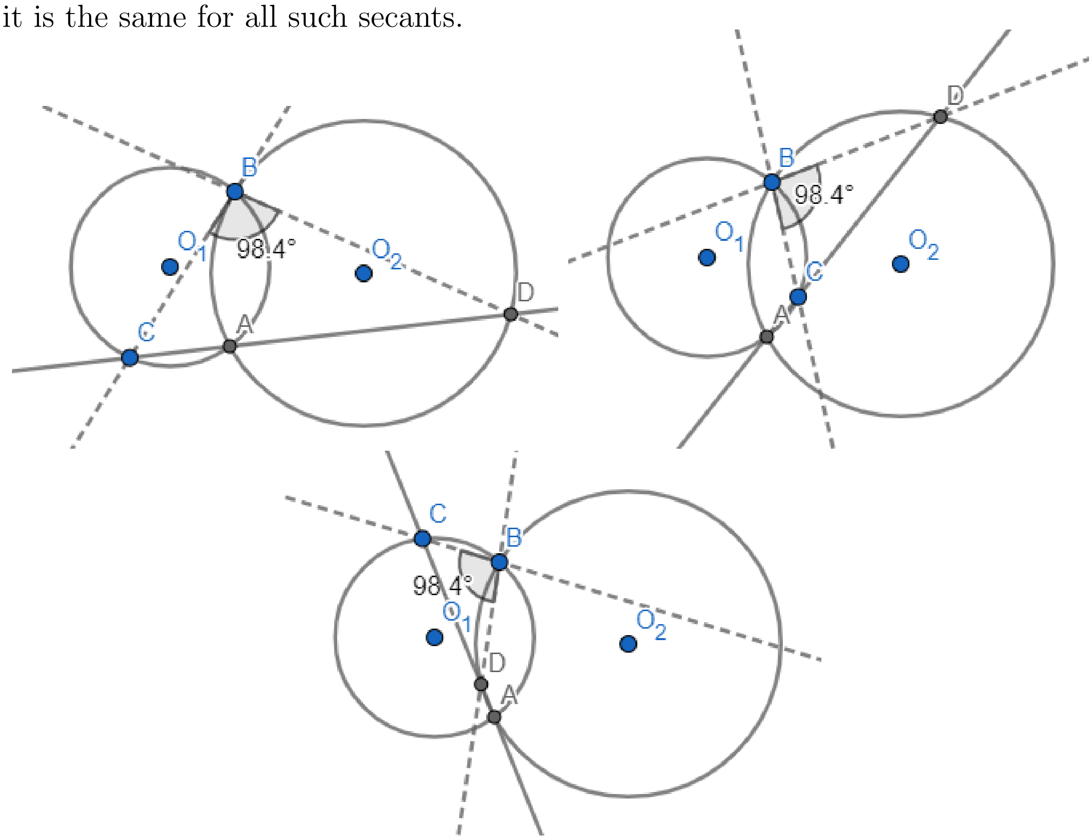
$$\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} > \frac{13}{24}$$

for all $n \geq 2$.

Problem 7.
Let $ABCD$ be points lying on a circle, in this order. Show that the opposite angles of the quadrilateral $ABCD$ add up to 180°.



Problem 8.
Two circles intersect at the points A and B, and through A, a secant is drawn intersecting the circles at the points C and D. Prove that the measure of the angle CBD is constant, i.e. it is the same for all such secants.



Problem 9.
Let PA and PB be two tangents to a circle drawn from the same point P, and let BC be a diameter. Prove that CA and OP are parallel.