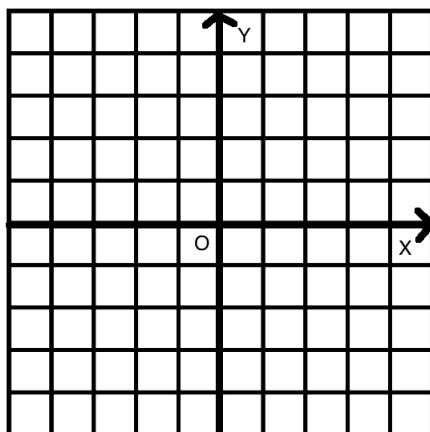


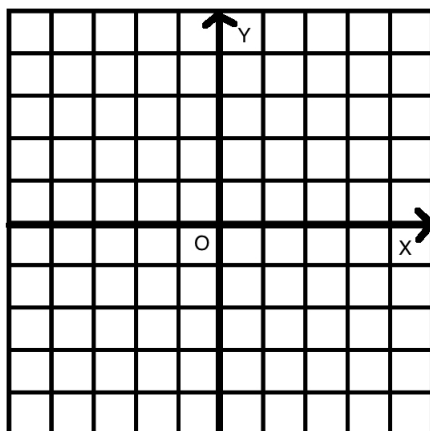
EXPLORING THE CITY OF DESCARTES - PART III

JUNIOR CIRCLE 04/17/2011

- (1) The streets in the city of Descartes run in the north-south (up-down) and east-west (right-left) directions only. Bob the taxidriver measures distances between two houses in the city by finding the smallest number of blocks he has to travel (both vertically and horizontally) to reach one of the houses from the other in the fastest possible way. (Remember that the streets run only in vertical and horizontal directions)
- (a) What is the distance between the points $(4, 3)$ and $(4, 7)$ according to Bob?



- (b) What is the distance between the point $(6, 5)$ and $(8, 5)$ according to Bob?



- (2) Bob's passenger wants to go from $(1, 1)$ to $(3, 3)$ as follows:
- 2 blocks down;
 - 2 blocks to the right;
 - 4 blocks up;

This can be described by the following code: $2 \downarrow, 2 \rightarrow, 4 \uparrow$.

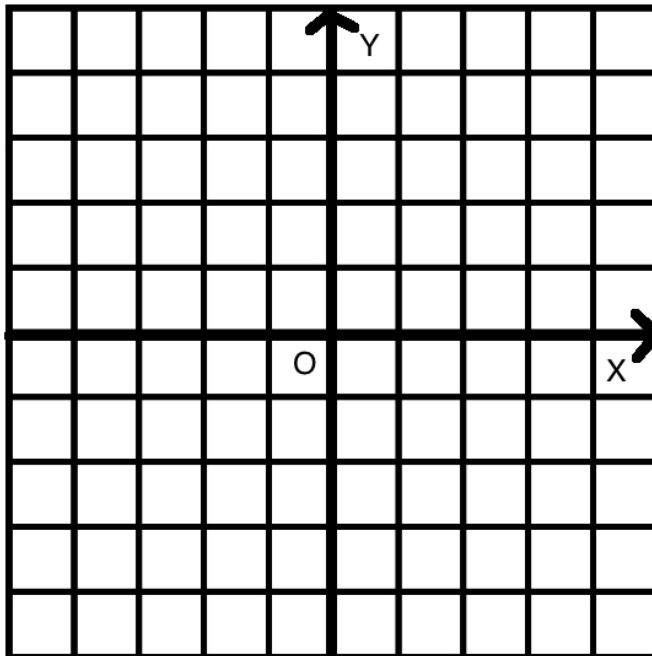
(a) Will Bob's passenger get to $(3, 3)$ if he starts at $(1, 1)$ and follows this plan?

(b) How many blocks total does this route take?

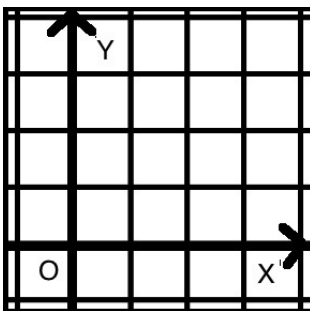
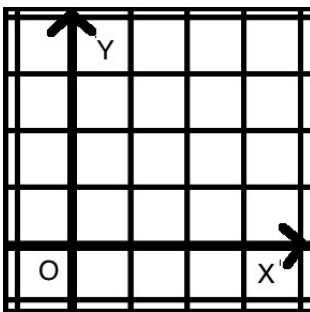
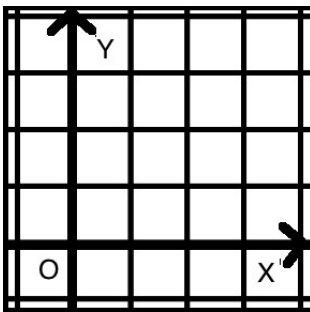
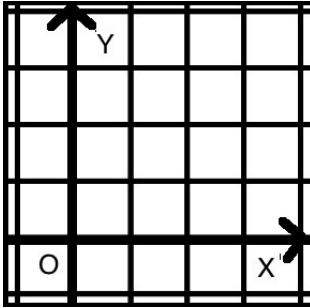
(c) Bob says that he can get from $(1, 1)$ to $(3, 3)$ by driving only 4 blocks. Can you draw a possible route? Describe it using the code above.

(d) Can you find a shorter route?

(e) What is Bob's distance from $(1, 1)$ to $(3, 3)$?

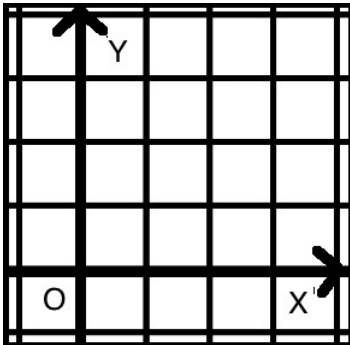


- (3) Find all possible routes from $(1, 1)$ to $(3, 3)$ that have length 4. Draw the route and describe each of the routes by a code: (*Note: Write the code next to the route you drew*)

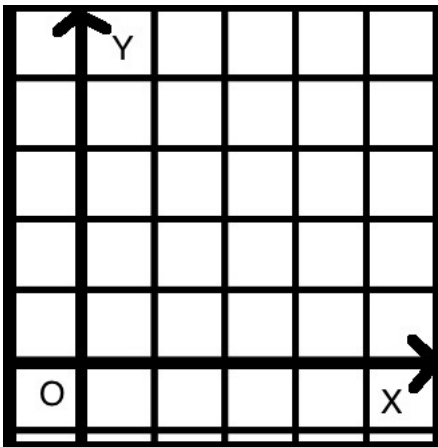


- Are there any other shortest routes?

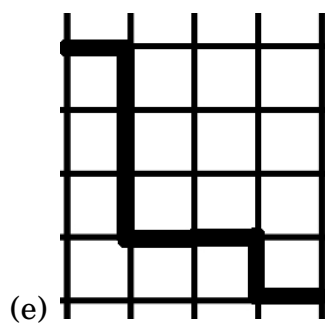
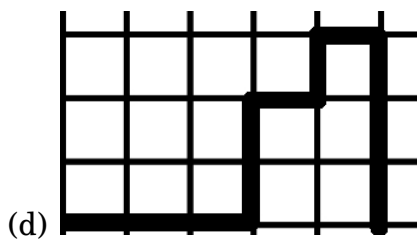
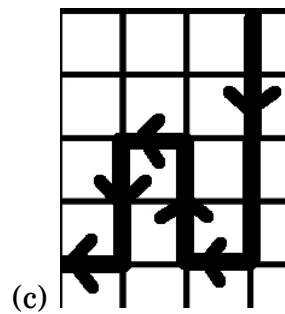
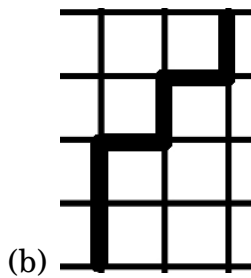
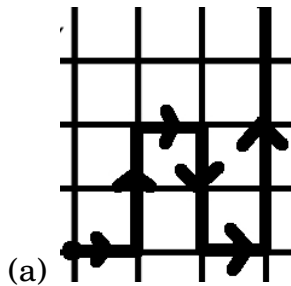
- (4) What is the distance between the points $(0, 0)$ and $(2, 3)$ according to Bob? Draw two possible routes that Bob can take from $(0, 0)$ to $(2, 3)$. Do they have the same length? Remember that Bob selects the *shortest* routes.



- (5) Bob's passenger wants to go from $(0, 0)$ to $(4, 5)$. He thinks that the distance between these two points is 13 because when he drives himself he uses the following route: $5 \rightarrow, 5 \uparrow, 1 \leftarrow$. Is he taking the shortest possible route? Give an example of a shorter route. Draw the route and write down the code for it.



- (6) The pictures below are fragments of routes between some points. Which of the following can *not* be a shortest route. Explain why and cross out the part of the route that makes it longer.



(7) The mayor wants to go from $(0, 0)$ to $(1000, 1000)$. His assistants proposed several routes that start as follows:

- $100 \uparrow, 53 \rightarrow, 27 \downarrow, 50 \rightarrow, 50 \uparrow \dots$
- $230 \leftarrow, 60 \uparrow, 80 \rightarrow, 330 \downarrow, 145 \rightarrow \dots$
- $500 \uparrow, 200 \rightarrow, 200 \uparrow, 300 \rightarrow, 300 \uparrow \dots$
- $400 \rightarrow, 200 \uparrow, 300 \rightarrow, 100 \downarrow, 350 \rightarrow \dots$
- $600 \uparrow, 145 \downarrow, 500 \rightarrow, 200 \leftarrow, 700 \uparrow \dots$

Assume the mayor wants to get there as fast as possible. What are the routes he should *not* take? Why?

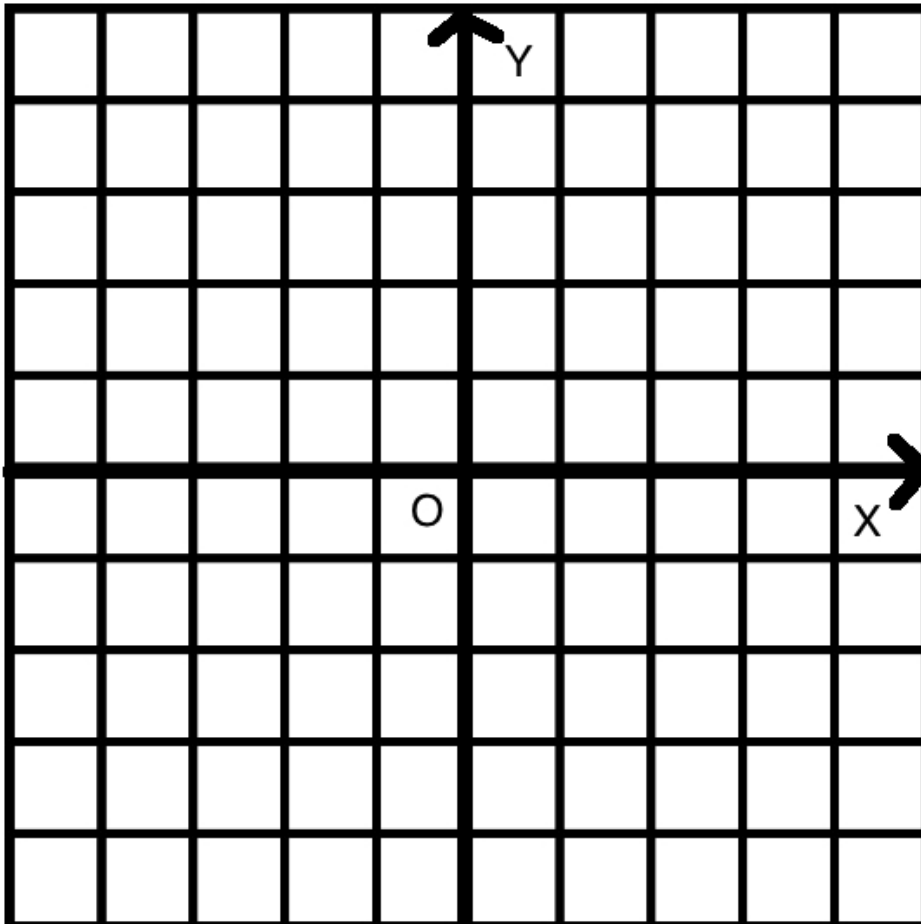
Select the best route and input the last segment.

(8) After working for a while, Bob decided to always go left (or right) as much as possible and then up (or down) as much as possible so his route consists of only two segments (horizontal, then vertical). Draw the routes that follow this idea between the following points and write the codes:

Starting Point	Ending point	Code	Bob's distance
$(3, 3)$	$(4, 6)$	$1 \rightarrow, 3 \uparrow$	
$(4, 4)$	$(-1, 2)$		
$(2, 1)$	$(5, 3)$		
$(5, 3)$	$(1, 1)$		
$(7, 5)$	$(1, 0)$		
$(0, 0)$	(a, b)		

(a) How is Bob's distance expressed in terms of the numbers in the code?

- (9) Bob starts at $O = (0, 0)$. He goes at the speed of one block per minute. (So it takes him 1 minute to go from $(0, 0)$ to $(1, 0)$, or from $(0, 0)$ to $(0, 1)$). Moreover, Bob *always drives away from O* .
- (a) Mark all the points that Bob can reach starting from O in exactly 1 minute by placing the Roman numeral **I** next to them. How many such points are there?
- (b) Mark all the points that Bob can reach starting from O in exactly 2 minutes by placing the Roman numeral **II** next to them. How many such points are there?
- (c) Mark all the points that Bob can reach starting from O in exactly 3 minutes by placing the Roman numeral **III** next to them. How many such points are there?
- (d) Describe the general pattern. Can you explain it?



- (10) Bob's friend Dan is a helicopter pilot. Dan can fly between any two points in the city along a straight line. So, Dan's distance is the usual distance between points.

We will compare distances between various points from Bob's and Dan's point's of view:

- (a) Give an example of a pair of points for which Bob's distance is the same as Dan's distance:
- (b) Give an example of a pair of points for which Dan's distance is shorter than Bob's distance:
- (c) Are there any pairs of points for which Bob's distance is smaller than Dan's distance? Why or why not?

