Counting and Pigeonhole

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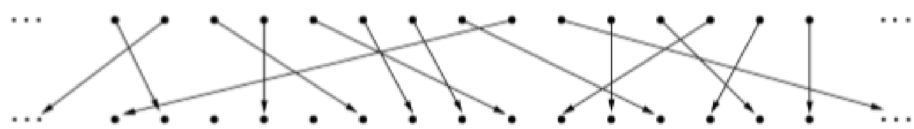
1. (AMC 10A 2003 #21, before stars & bars was well known) Pat is to select six cookies from a tray containing only chocolate chip, oatmeal, and peanut butter cookies. There are at least six of each of these three kinds of cookies on the tray. How many different assortments of six cookies can be selected?

2. (BAMO 12 2012 #2) Two infinite rows of evenly-spaced dots are aligned as in the figure below. Arrows point from every dot in the top row to some dot in the lower row in such a way that:

   - No two arrows point at the same dot.
   - No arrow can extend right or left by more than 1006 positions.

   Show that at most 2012 dots in the lower row could have no arrow pointing to them.



3. S is a set of $n+1$ integers. Show that there exist distinct a, b in S such that $a-b$ is a multiple of n.

4. Show that in any group of $n \geq 2$ people there are two who have the identical number of friends in that group.

5. Assume that 101 distinct points are placed in a square $10x10$ such that no three of them lie on a line. Prove that we can choose three of the given points that form a triangle whose area is at most 1.

6. A set $S \subseteq \{2, 3, 4, 5, \cdots\}$ is called composite if no two elements of S are relatively prime. Find the maximal integer k for which the following proposition holds: Every composite set S with at least 3 elements has an element greater than or equal to $k|S|$.