

LAMC Week 7: Number Theory Contest

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1. Let a, b, c be positive integers. Prove that

$$\frac{\text{lcm}(a, b, c)^2}{\text{lcm}(a, b)\text{lcm}(b, c)\text{lcm}(a, c)} = \frac{\text{gcd}(a, b, c)^2}{\text{gcd}(a, b)\text{gcd}(a, c)\text{gcd}(b, c)}$$

2. Let $f_0 = 0, f_1 = 1$, and $f_{n+1} = f_n + f_{n-1}$. Prove that if $s \geq 1, t \geq 0$, then $f_{s+t} = f_{s-1}f_t + f_s f_{t+1}$.
3. Prove that the sum $\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is never an integer.
4. Let k a positive integer. Show that there exists n so that $\varphi(n) = k!$.
5. Let a, b be positive integers. Then, show that $\frac{2a^2-1}{b^2+2}$ is not an integer.