LAMC Week 6: Solving Diophantine Equations

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17 November 2019

1. Find all pairs of positive integers x, y such that

$$\frac{1}{x} + \frac{1}{y} = \frac{3}{14}$$

2. (Balkan MO) Show that the equation $x^5 - y^2 = 4$ has no solutions in integers.

3. (From week 1) A rectangular chocolate bar forms an $m \times n$ grid of squares, with m, n positive integers. If we cut the chocolate bar along the diagonal, how many squares are divided into two parts of equal size?

4. (AIME I 2015) There is a prime number p such that $16p + 1$ is the cube of a positive integer. Find p.

5. (IMO 1990) (From last week) Determine all integers $n > 1$ such that

$$\frac{2^n + 1}{n^2}$$

is an integer.

6. (USAMO 2005) (Easier than it looks) Prove that the system

$$x^6 + x^3 + x^3 y + y = 147^{157}$$
$$x^3 + x^3 y + y^2 + y + z^9 = 157^{147}$$

has no solutions in integers x, y, and z.

7. (BAMO 2011) (Hard) Do there exist $n \in \mathbb{Z}$ and $0 \leq a, b, c, d \leq n$, such that

$$\binom{n}{a}, \binom{n}{b}, \binom{n}{c}, \binom{n}{d}$$

are distinct and

$$\binom{n}{a} = 2\binom{n}{b}, \quad \binom{n}{c} = 2\binom{n}{d}?$$