LAMC Week 5: Lifting the Exponent lemma (Solutions)

Jacob Zhang, Shend Zhjeqi

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Problem 1: We get mod 3 that n is even. Using LTE we get $k - 1 \leq v_3(n)$.

Problem 2: If $p = 2$ we are done. If p odd, by LTE (v_5) we get either $n = 1$ or otherwise $n = 2$ and p=5 which fails. So, n =1.

Problem 3: n is odd hence we can use LTE and let p be the minimal prime greater than 3 dividing $2^n + 1$. Then, using a basic property of congruences and Fermat we get p =3. However, by LTE we show (n,3)=1, hence n=1 or $n = 3$.

Problem 4: Choose $p < q$ both different from 3. Use Fermat and property of congruences to get contradiction. $p = q$ again one of them is 3.

Problem 5: Show a is odd and then use LTE $a \leq 1 + v_3(a)$.

Problem 6: Let $a = p_1 p_2 \cdots p_n$ and $b = 2^a + 1$. It is sufficient to prove that b has at least $2n$ prime divisors. This is indeed true, because Zsigmondy's theorem for sums says that as a is not divisible by 3, the divisor $2^d + 1$ introduces a new prime for every divisor $d|a$. As a has $2n$ divisors, b has at least $2n$ prime divisors, which is much bigger than the required $2n$ divisors.