LAMC Week 5: Lifting the Exponent lemma

Jacob Zhang, Shend Zhjeqi

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1. Let k be a positive integer. Find all positive integers n such that

$$3^k | 2^n - 1.$$

2. (Ireland 1996) Let p be a prime number, and a and n positive integers. Prove that if $2^p + 3^p = a^n$, then $n = 1$.

3. (IMO 1990) Determine all integers $n > 1$ such that

$$\frac{2^n + 1}{n^2}$$

is an integer.

4. Find all primes p, q such that $\frac{(5^p - 2^p)(5^q - 2^q)}{pq}$ is an integer.

5. Find all positive integers a such that $\frac{5^a + 1}{3^a}$ is a positive integer.

6. (IMO Shortlist 2002) Let $p_1, \cdots, p_n$ be distinct primes greater than 3. Show that $2^{p_1 \cdots p_n} + 1$ has at least 4^ndivisors.