

# LAMC Week 4: NT Problem Session

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1. Let  $F_n$  be the  $n$ -th Fibonacci number. Prove that  $F_1^2 + \cdots + F_n^2$  is divisible by  $F_n$ .
2. Show there are infinitely many primes of the form  $4k + 3$ .
3. For any given positive integer  $n$ , find the number of congruence classes  $x \pmod{n}$  that satisfy

$$x^2 \equiv 1 \pmod{n}.$$

4. Let  $p, q$  be two distinct primes. Show that  $\gcd(5^p - 2^p, 5^q - 2^q) = 3$ .
5. Assume that  $n$  is an odd positive integer. Then,  $n$  divides  $2^{(n-1)!} - 1$ .
6. (Bonus) Show that there are infinitely many primes of the form  $4k + 1$ .