

LAMC Week 3: Euler's φ function

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1. Find the last two digits of 2^{2019} .
2. Let f, g be multiplicative functions, i.e. $f(mn) = f(m)f(n)$ when $\gcd(m, n) = 1$, and same for g .
 - (a) Prove that $f(n)g(n)$ and $f(n)/g(n)$ are also multiplicative functions.
 - (b) Prove that

$$h(n) = \sum_{d|n} f(d)g\left(\frac{n}{d}\right)$$

is also a multiplicative function. (More on this in a future class!)

- (c) (CCA Math Bonanza) Compute

$$\sum_{k=1}^{420} \gcd(k, 420)$$

3. (PuMAC) Find the last three digits of

$$2008^{2007^{2006 \cdots 2^1}}$$

4. (USAMO 1991) Prove that for all n the sequence $2, 2^2, 2^{2^2}, \dots$ is eventually constant modulo n .
5. (USA(J)MO 2011) Consider the assertion that for each positive integer $n \geq 2$, the remainder upon dividing 2^{2^n} by $2^n - 1$ is a power of 4. Either prove the assertion or find (with proof) a counterexample.