Lesson 8: Games and Geometry III

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Problem 1.
Show that in a game of tic-tac-toe on an infinite board the second player does not have a winning strategy. In infinite tic-tac-toe one needs 5 in a row to win.

Problem 2.
Two players are playing a game at night on the streets of the Candy Kingdom. The streets of the Candy Kingdom make a rectangular grid. Every turn consists of finding a not yet lit intersection, and putting a projector there, which lights up everything to the top and right of itself (including the intersection it is on). The person, after whose move the whole kingdom is lit for the first time loses. Who has a winning strategy?

Problem 3.
Find the geometric locus of centers of circles described by a given radius and tangent to a given circle(consider two cases: of external and internal tangency).

Problem 4.
Prove that the shortest segment joining two non-intersecting circles lies on the line of centers.

(Hint: Apply the triangle inequality)

Problem 5.
Suppose n points are marked on the plane, where $n \geq 9$. It is known that for any 9 of the points one can draw two circles so that all 9 points lie on those circles. Show that it is possible two draw two circles so that all n points lie on them.